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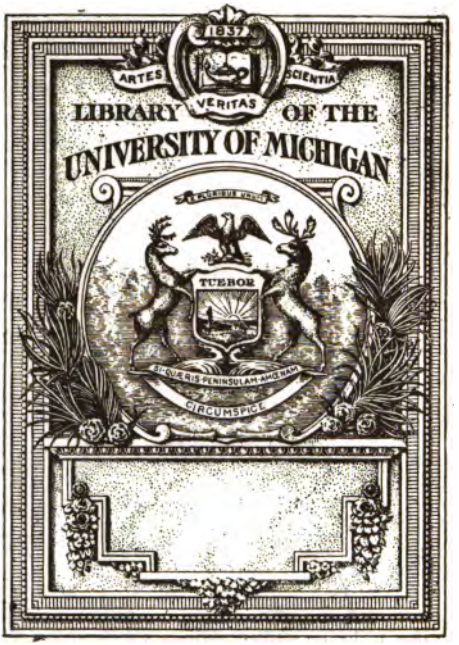
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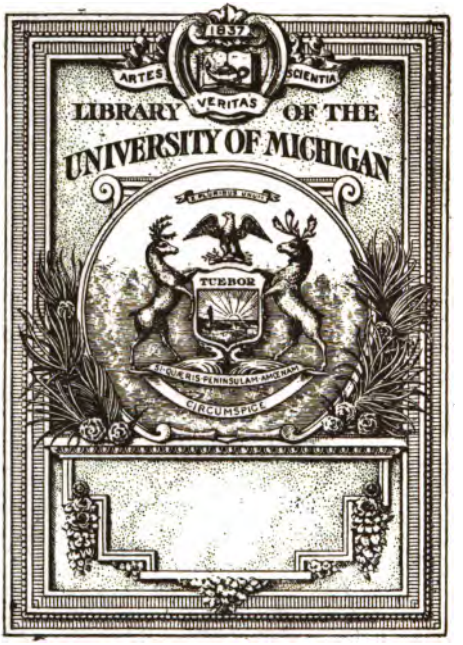
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WENTWORTH-SMITH MATHEMATICAL SERIES

VOCATIONAL ALGEBRA

BY

GEORGE WENTWORTH

AND

DAVID EUGENE SMITH

GINN AND COMPANY

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PREFACE

The time has arrived when algebraic language has such a well-defined place in trade journals, artisans' manuals, and handbooks of business, that the workman in the shop and the business man in the office have each a practical need to interpret it. The growth of industrial and commercial classes, one of the most significant features of our present work in education, helps to create such a need, and the shop itself is not far behind the school in making it known.

But when we consider the amount of algebra actually required to read with intelligence a trade journal or a workman's manual, or to throw light upon the solution of any useful problem of arithmetic, we find that the range of work is not extensive and that the difficulties of the subject are few. We are struck by the fact that the demand is generally limited to two things, — the ability to understand and manipulate a formula, and the power to solve a simple equation in one unknown quantity. It is evident that this is only a small part of algebra, and that one who has mastered merely this portion has still a rich field before him, and that much additional power will come from a further study of the subject. Nevertheless, if circumstances are not favorable to this further study, it is a worthy educational policy that offers to the student the simplest working tools of the subject.

In attempting to meet the demand for the essentials of the algebra that is needed in the shop and in commerce, the authors have felt free to depart from the conventional arrangement of matter and from the traditional applications of the subject. Since the work is, as its title indicates, merely a summary of the elementary vocational aspects of

algebra, not leading directly to any subsequent work in the subject, the criterion of sequence has been largely the immediate needs of the student. When we investigate these needs we see that the first demand is met most naturally by considering the rules of mensuration already familiar to the student from his arithmetic, these now being stated in algebraic language and the resulting formulas evaluated. As soon as this language is understood, the next need is the ability to solve a simple equation, including the usual manipulation of the formulas. The negative number, which enters into so many formulas, is then considered, and is followed by such algebraic operations with integers and fractions as are indispensable in the treatment of the formulas of trade, in the solution of such equations as are liable to be met by every one, and in an elementary discussion of proportion.

For pupils using Book III of the authors' arithmetics this work is unnecessary, since it is substantially included in that text. The opportunity that comes from publishing it in separate form to meet the needs of other pupils has permitted a slight extension of the treatment and the addition of a few features for which there is not time in the usual course in arithmetic. The authors have, however, kept strictly to their original purpose, of presenting only the vocational phase of the subject, with such drill in the manipulation of algebraic expressions as is essential to this work, and with the formula and the equation continually reviewed and made the leading feature in every chapter.

Suggestions for the improvement of the work will be thankfully received from all who, with the authors, are interested in the momentous question of meeting all well-considered demands for the improvement of mathematical instruction.

GEORGE WENTWORTH
DAVID EUGENE SMITH

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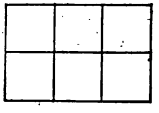
VOCATIONAL ALGEBRA

CHAPTER I

ALGEBRAIC EXPRESSIONS

1. Algebraic Forms. In arithmetic we learned that the area of a rectangle equals the product of its base and height. We express this more briefly in *algebraic* form as follows:



$$a = bh.$$

Here we have let a stand for the word *area*, b for the word *base*, and h for the word *height*. When we write bh it indicates the *product* of b and h.

In arithmetic 25 means $20 + 5$, but in algebra bh means $b \times h$. We also write $b \cdot h$ for this product, so that $b \times h$, $b \cdot h$, and bh all mean the same.

When two letters are written side by side in algebra, with no sign between them, the product of their numerical values is to be taken.

Thus, if $b = 4$ and $h = 2$, then $a = bh = 4 \times 2 = 8$; if $b = 7\frac{1}{2}$ and $h = 2\frac{1}{4}$, then $bh = 7\frac{1}{2} \times 2\frac{1}{4} = 16\frac{7}{8}$; if $b = 4.02$ and $h = 0.7$, then $bh = 4.02 \times 0.7 = 2.814$.

In writing algebraic forms it is not customary to express denominations, like feet or inches. In arithmetic we would say that if $b = 3$ in. and $h = 2$ in., a would equal 3×2 sq. in., or 6 sq. in., but in algebra we omit the inches and square inches.

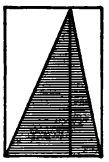
When we speak of the product of two lines, this is a brief way of expressing the product of their *numerical values*.

2. Parallelogram. We have learned that the area of a parallelogram equals the product of its base and height. We may therefore express this in algebraic form thus: $a = bh$.

For the triangle T may be cut off and placed at X, the parallelogram then becoming a rectangle.

If $b = 5$ and $h = 3\frac{1}{2}$, then $a = bh = 5 \times 3\frac{1}{2} = 17\frac{1}{2}$. If b and h represent inches, then a represents square inches.

3. Triangle. We have learned that the area of a triangle equals half the product of its base and height. This is expressed in algebraic form thus: $a = \frac{1}{2}bh$.



For the triangle may be cut as here shown so that it is seen to be half of the rectangle of base b and height h.

If $b = 7$ and $h = 10\frac{1}{2}$, then $a = \frac{1}{2}bh = \frac{1}{2}$ of $7 \times 10\frac{1}{2} = 36\frac{3}{4}$.

Exercise 1. Parallelograms and Triangles

Given $a = bh$, find the value of a when:

1. $b = 7.2$, $h = 3.4$.
2. $b = 8.9$, $h = 4.7$.
3. $b = 127\frac{3}{4}$, $h = 27.4$.
4. $b = 22.75$, $h = 14\frac{3}{4}$.

Given $a = \frac{1}{2}bh$, find the value of a when:

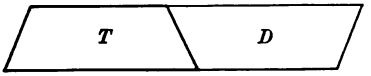
5. $b = 42$, $h = 25$.
6. $b = 36$, $h = 19$.
7. $b = 56$, $h = 27$.
8. $b = 23.8$, $h = 4.75$.
9. $b = 41.2$, $h = 19.65$.
10. $b = 33.3$, $h = 24.50$.

11. A playground is l feet long and w feet wide. Find the area in square feet. What is the number of square feet when $l = 124$ and $w = 62\frac{1}{2}$?

12. A triangular space at the end of a house, under the roof, is f feet wide and h feet high. What is the area in square feet?

4. Trapezoid. We have learned that the area of a trapezoid equals half the product of the sum of the two parallel sides multiplied by the height.

For an equal trapezoid may be turned over and put down by the side of the first one, as here shown. The whole figure, or twice the trapezoid, then equals a parallelogram whose base is the sum of the parallel sides of the trapezoid. The trapezoid is therefore half this parallelogram.



We indicate this in algebraic form as follows:

$$a = \tfrac{1}{2}(b + b')h,$$

where b and b' ("b prime") are the two parallel sides, usually called the *bases*, and h is the height.

The parentheses show that b and b' are to be added before being multiplied by h. The operations indicated within parentheses are always performed first. Thus if $b = 6$, $b' = 5$, and $h = 4$, we have $a = \frac{1}{2}(b + b')h = \frac{1}{2}(6 + 5) \times 4 = \frac{1}{2}$ of $11 \times 4 = 22$.

Exercise 2. Trapezoids

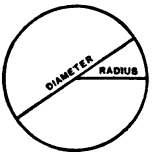
Given $a = \frac{1}{2}(b + b')h$, find the value of a when:

1. $b = 6$, $b' = 4$, $h = 3$.
2. $b = 9$, $b' = 5$, $h = 7$.
3. $b = 11$, $b' = 8$, $h = 6$.
4. $b = 13$, $b' = 7$, $h = 7$.
5. $b = 19$, $b' = 9$, $h = 9$.
6. $b = 21$, $b' = 9.5$, $h = 7.2$.
7. $b = 32$, $b' = 16$, $h = 19$.
8. $b = 28$, $b' = 9.8$, $h = 8.6$.
9. $b = 4.9$, $b' = 3.4$, $h = 1\frac{1}{2}$.
10. $b = 4\frac{3}{4}$, $b' = 3\frac{1}{4}$, $h = 2\frac{1}{2}$.

11. The lower base of a trapezoid is 10 in. and the altitude and upper base are each 5 in. Find the area.

12. A field is in the form of a trapezoid, with bases x and y, and with altitude z. What is the area of the field? How many square rods are there in the field if $x = 30$ rods, $y = 26$ rods, and $z = 24$ rods?

5. Circumference. We have learned that the circumference of a circle equals nearly 3.1416 × the diameter. In mathematics the number 3.1416, approximately $3\frac{1}{7}$, is represented by the Greek letter π (pronounced *pī*). We may therefore express this law as follows:



$$c = \pi d,$$

where c stands for circumference, d for diameter, and π for $3\frac{1}{7}$, or 3.1416, nearly.

Since the diameter equals twice the radius, we may write $2r$ for d, and have $c = \pi \times 2r$, or $c = 2\pi r$.

Thus if $d = 7$, and we take $3\frac{1}{7}$ as the value of π, we have

$$c = \pi d = 3\tfrac{1}{7} \times 7 = 22.$$

If $r = 5$, we have $c = 2\pi r = 2 \times 3\frac{1}{7} \times 5 = 31\frac{3}{7}$.

Exercise 3. Circumferences

Given $c = \pi d = 2\pi r$, and taking $\pi = 3\frac{1}{7}$, find c when:

1. $d = 14$.	**4.** $d = 3.5$.	**7.** $r = 7$.	**10.** $r = 6.3$.
2. $d = 21$.	**5.** $d = 4.9$.	**8.** $r = 3.5$.	**11.** $r = 9.1$.
3. $d = 28$.	**6.** $d = 7.7$.	**9.** $r = 5.6$.	**12.** $r = 10\frac{1}{2}$.

Taking $\pi = 3.1416$, find c when:

13. $d = 10$.	**15.** $d = 50$.	**17.** $r = 30$.	**19.** $r = 2\frac{1}{2}$.
14. $d = 20$.	**16.** $d = 25$.	**18.** $r = 40$.	**20.** $r = 7.5$.

21. If a globe has a diameter of 14 in., what is the circumference?

22. If you describe a circle with a radius of 12 in., how many inches will there be in the circumference?

23. A workman measures the diameter of a steel shaft and finds it to be $3\frac{7}{16}$ in. What is the circumference?

6. Area of a Circle. We have learned that

1. The area of a circle equals half the product of the circumference and radius.

2. The area of a circle equals 3.1416 times the square of the radius.

We may express the first of these laws as follows:

$$a = \tfrac{1}{2}\, cr.$$

To express the second of these laws, since $c = 2\,\pi r$ (§ 5), we may put $2\,\pi r$ for c, and then we have $a = \frac{1}{2}$ of $2\,\pi rr$, or, writing r^2 ("r square") for rr,

$$a = \pi r^2.$$

Thus if $r = 10$, we have $a = \pi r^2 = 3.1416 \times 10^2 = 314.16$.

Exercise 4. Area of a Circle

Given $a = \frac{1}{2}\, cr$, find a when:

1. $r = 5$, $c = 31.416$.
2. $r = 10$, $c = 62.832$.
3. $r = 20$, $c = 125.664$.
4. $r = 2\frac{1}{2}$, $c = 15.708$.
5. $r = 25$, $c = 157.08$.
6. $r = 50$, $c = 314.16$.

Given $a = \pi r^2$, and taking $\pi = 3\frac{1}{7}$, find a when:

7. $r = 7$.
8. $r = 3\frac{1}{2}$.
9. $r = 14$.
10. $r = 2.1$.
11. $r = 2.8$.
12. $r = 35$.
13. $r = 4.9$.
14. $r = 7.7$.

Given $a = \pi r^2$, and taking $\pi = 3.1416$, find a when:

15. $r = 5$.
16. $r = 10$.
17. $r = 20$.
18. $r = 50$.

19. How many square feet in the area of a circle whose radius is 2 ft.? (Take $\pi = 3.1416$.)

20. How many square feet in the circular base of a water tank whose radius is 15 ft.? (Take $\pi = 3.1416$; then take $\pi = 3\frac{1}{7}$, and find the difference in the results.)

7. Cylinder. We have found that the volume of a cylinder equals the product of its base and height; that is, $v = bh$, where v stands for volume, b for base, and h for height.

But since $b = \pi r^2$ (§ 6), we may put πr^2 for b, and then we have

$$v = \pi r^2 h.$$

8. Formula. We therefore see that algebra enables us to put a long rule of arithmetic into a brief form. An algebraic form of this kind is called a *formula.*

When we find the value of an algebraic expression by putting numbers in place of the letters, we are said to *substitute* numbers for the letters and to *evaluate* the formula.

Exercise 5. Formulas

Express the following statements in formulas:

1. The lateral area of a cylinder equals the product of the circumference and height. (Use l for lateral area, c for circumference, and h for height.)

2. The lateral area of a cylinder equals 3.1416 times the diameter multiplied by the height.

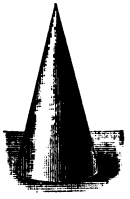
3. The lateral area of a cylinder equals twice 3.1416 times the radius multiplied by the height. (Use r for radius, and in general in a formula use the initial letter for a word.)

4. The volume of a prism equals the product of the base and height.

5. The volume of a pyramid equals one third the product of the base and height.

6. The volume of a cone equals one third the product of 3.1416 times the square of the radius of the base multiplied by the height.

7. The formula for the lateral surface of a cone in terms of the circumference of the base and k the distance from the vertex to the circumference of the base is $l = \frac{1}{2} kc$. Write this as an ordinary sentence.



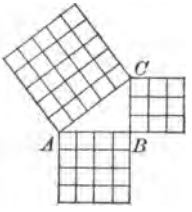
8. The formula for the lateral surface of a cone in terms of the radius of the base and the distance from the vertex to the circumference of the base is $l = \pi kr$. Write this as an ordinary sentence.

9. The formula for the surface of a sphere is $s = 4\pi r^2$. Write this as an ordinary sentence.

10. The formula for the volume of a sphere is $v = \frac{4}{3}\pi r^3$, where r^3 ("r cube") means rrr. Write this as an ordinary sentence.

11. The square on the hypotenuse of a right triangle is stated in arithmetic to be equal to the sum of the squares on the other two sides. Letting $AB = x$, $BC = y$, and $AC = z$, we have $z^2 = x^2 + y^2$, or $z = \sqrt{x^2 + y^2}$. Suppose $x = 4$ and $y = 3$, find the value of z.

(The expression $\sqrt{x^2 + y^2}$ means that we are to square x and to square y, then to add these results, and then to extract the square root of the sum.)



12. Given $z = \sqrt{x^2 + y^2}$, find the value of z when $x = 24$ and $y = 32$.

13. In the same formula find the value of z when $x = 36$ and $y = 27$.

14. In geometry it is shown that the sum of all the angles of a polygon of n sides is $(n - 2)\ 180°$. What is the sum of the angles in a triangle? in a quadrilateral? in a polygon of five sides? in a polygon of twenty sides?

Evaluate the following by substituting the given values:

15. $l = dh$, where $d = 16.4$ and $h = 7$.

16. $l = ch$, where $c = 17.6$ and $h = 19.3$.

17. $z = \sqrt{x^2 + y^2}$, where $x = 33$ and $y = 44$.

18. $v = bh$, where $b = 147.6$ and $h = 78.4$.

19. $v = \frac{1}{3} bh$, where $b = 275.4$ and $h = 124.2$.

20. $v = \frac{1}{3} \pi r^2 h$, where $r = 9$, $h = 7$, and $\pi = 3\frac{1}{7}$.

21. $l = \frac{1}{2} dc$, where $d = 10$ and $c = 31.416$.

22. $l = \pi kc$, where $k = 7$, $c = 12$, and $\pi = 3\frac{1}{7}$.

23. $l = \pi kr$, where $k = 5$, $r = 7$, and $\pi = 3\frac{1}{7}$.

24. $s = 4 \pi r^2$, where $r = 10$ and $\pi = 3.1416$.

25. $v = \frac{4}{3} \pi r^3$, where $r = 3$ and $\pi = 3.1416$.

26. If an automobile has a constant velocity of 10 mi. an hour, how far will it go in 4 hr.? Write a formula for d, the distance it will go in t hr. at v mi. per hour.

27. What is the volume of a box 9 in. long, 8 in. wide, and 6 in. deep? Write a formula for v, the volume of a box l in. long, w in. wide, and d in. deep.

28. The formula for the radius in terms of the circumference is $r = \frac{c}{2\pi}$. Write this as an ordinary sentence.

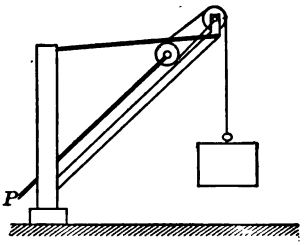
29. If it is c in. around an iron pipe, the diameter is $\frac{c}{\pi}$ in.; that is, $d = \frac{c}{\pi}$. Write this as an ordinary sentence.

30. The circumference of a water pipe is 22 in. Using the formula in Ex. 29, and taking $3\frac{1}{7}$ as the value of π, find the diameter.

31. It is shown in geometry that the area of a triangle whose sides are a, b, and c, is $\sqrt{s(s-a)(s-b)(s-c)}$, where s stands for $\frac{1}{2}(a + b + c)$. Find the area of the triangle whose sides are 6, 8, and 10.

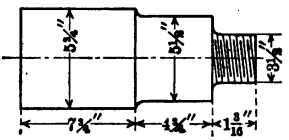
32. A man who runs a stationary engine for hoisting iron reads, in a book about engines, that if s is the area of the outside shell of his boiler and h is the heating surface, then $h = \frac{2}{3} s$. What does this mean? What is the value of h if $s = 120\frac{5}{16}$ sq. ft.?

33. A foreman of a shop has a hoist like this for lifting heavy weights. He reads in a trade journal that in order to lift a weight W, the power P must be such that $P = 2W$. What power must his engine apply to the cable so as to lift a weight of 15,000 lb.? What weight could be lifted with a power of 75.4 T.?



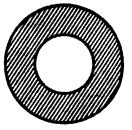
34. If a cubic inch of steel weighs 0.28 lb., what will be the weight of a steel cylinder d in. in diameter and l in. long, it being known that the volume is $v = \frac{1}{4} \pi d^2 l$? What will it be if $d = 1$, $l = 7$, and $\pi = 3\frac{1}{7}$?

35. A machinist is making a "crank pin" (a kind of bolt) for an engine, according to this drawing. He considers it as weighing the same as three steel cylinders having the diameters and lengths in inches as here shown, where $7\frac{3}{4}''$ means $7\frac{3}{4}$ in. He has this formula for the weight (w) of a steel cylinder where d is the diameter and l is the length: $w = 0.07\, \pi d^2 l$. (Taking $\pi = 3\frac{1}{7}$, find the weight of the pin.)



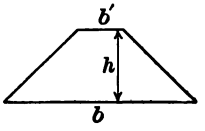
36. The volume of a cylinder being represented by $v = \pi r^2 h$, what will the water in a cylindrical water tank weigh when $r = 9$ ft., $h = 10$ ft., $\pi = 3\frac{1}{7}$, and the weight of 1 cu. ft. of water is $62\frac{1}{2}$ lb.?

37. This figure represents the cross section of a water pipe. If the radius of the outer circle is r and that of the inner circle is r', then the area of the shaded portion (the iron) is found by this formula: $a = \pi(r^2 - r'^2)$. Find the value of a if $r = 4$ and $r' = 3$. (Use $\pi = 3\frac{1}{7}$.)



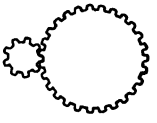
38. The weight of a hollow steel pillar whose length is l, and the radii of whose outer and inner circles are r and r', is $483\, l\pi(r^2 - r'^2)$ pounds, where l, r, and r' are expressed in feet. Find the weight when $l = 12$, $r = \frac{1}{3}$, $r' = \frac{1}{6}$, and $\pi = 3\frac{1}{7}$.

39. If this represents the cross section of a railway embankment l ft. long, h ft. high, b ft. wide at the bottom, and b' ft. wide at the top, the number of cubic feet in the embankment is represented by $v = \frac{1}{2} lh(b + b')$. Find the volume if $l = 300$, $h = 8$, $b = 60$, and $b' = 28$.

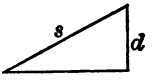


40. A cast-iron bar is l ft. long, and its cross section is a square e in. on a side. Write the formula for its volume in cubic inches; in cubic feet. If $l = 6$ and $e = 2$, what will the bar weigh, allowing 0.27 lb. to the cubic inch?

41. Two cogwheels, one having 9 cogs and the other 27, are fitted together. How many times will the smaller wheel turn for each turn of the larger? How many times, if the larger has a cogs and the smaller b cogs?



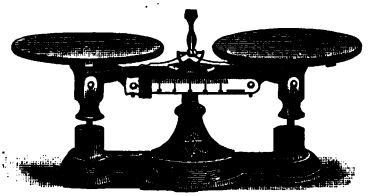
42. If a barrel weighing w lb. is rolled up an incline s ft. long, to a point d ft. high, there is exerted a power of $\frac{wd}{s}$; that is, $p = \frac{wd}{s}$. How much power must be used to roll a 200-pound barrel up a 10-foot incline to a height of 4 ft.?



CHAPTER II

EQUATIONS

9. The Equation. If we take a pair of scales, they will exactly balance if they are empty. If we add 3 oz. to each side, they will still balance. If we multiply each of these 3 oz. by 6, they will still balance. If we divide the weights by 2, they will still balance. If we tie to each side a balloon that pulls up 10 lb., they will still balance.



An expression of equality between two quantities is called an *equation.*

For example, $x + 5 = 7$. Here we see that $x = 2$.

10. The Principles of the Equation. We have seen that

(1) *If equals are added to equals, the results are equal.*

(2) *If equals are subtracted from equals, the results are equal.*

(3) *If equals are multiplied by equals, the results are equal.*

(4) *If equals are divided by equals, the results are equal.*

11. Solving an Equation by Subtracting. If we have

$$x + 5 = 7,$$

we may subtract 5 from these equals (§ 10), and then

$$x = 7 - 5 = 2.$$

Exercise 6. Equations involving Subtraction

Solve the following:

1. $x + 6 = 10.$
2. $x + 9 = 15.$
3. $x + 7 = 32.$
4. $x + 12 = 70.$
5. $x + 15 = 69.$
6. $x + 27 = 52.$
7. $x + 29 = 101.$
8. $x + 37 = 105.$
9. $x + 48.5 = 100.$
10. $x + 62\frac{3}{4} = 127.$

11. What number added to 11 equals 39?

12. What number added to 37 equals 92?

13. What number added to $42\frac{7}{8}$ equals $64\frac{3}{4}$?

14. If $63\frac{4}{5}$ is added to a certain number, the sum is 91. What is the number?

15. If $29\frac{7}{8}$ is added to a certain number, the sum is $42\frac{1}{2}$. What is the number?

16. If the sum of 29.42 and a certain number is 83.41, what is the number?

17. What number added to πr^2 makes $\pi r^2 + 5$? makes $\pi r^2 + 3$? makes 25?

18. From the formula $a = b + c$ find the value of c in terms of a and b. What does c equal when $a = 31.4$ and $b = 17.6$?

19. From the formula $a = \pi r^2 + c$ find the value of c in terms of the other letters. What does c equal when $a = 200$, $\pi = 3\frac{1}{7}$, and $r = 7$?

20. From the formula $h^2 = a^2 + b^2$ find the value of b^2 in terms of h and a. What does b^2 equal when $h = 20$ and $a = 16$?

21. From the formula $s = 2\pi r^2 + c$ find the value of c in terms of the other letters. What does c equal when $s = 400$, $\pi = 3\frac{1}{7}$, and $r = 7$?

12. Solving an Equation by Dividing. If we have the equation

$$3x = 27,$$

we may divide these equals by 3 (§ 10), and obtain

$$x = 9.$$

If we have the equation

$$3x + 7 = 34,$$

we may first subtract 7 from these equals, and obtain

$$3x = 27.$$

We may then divide by 3, and we find that

$$x = 9.$$

Exercise 7. Equations involving Division

Solve the following:

1. $3x = 30.$
2. $3x = 72.$
3. $5x = 95.$
4. $6x = 96.$
5. $7x = 91.$
6. $8x = 104.$
7. $9x = 153.$
8. $17x = 187.$
9. $23x = 483.$
10. $3.8x = 34.2.$
11. $28.6x = 20.02.$
12. $16\frac{2}{3}x = 316\frac{2}{3}.$
13. $14.4x = 100.8.$
14. $27.9x = 251.1.$
15. $1254x = 7524.$
16. $3x + 5 = 38.$
17. $3x + 7 = 106.$
18. $4x + 1 = 121.$
19. $5x + 6 = 131.$
20. $6x + 9 = 339.$
21. $7x + 8 = 106.$
22. $9x + 11 = 191.$
23. $12x + 27 = 171.$
24. $17x + 13 = 200.$
25. $2.9x + 29 = 58.$
26. $326x + 9 = 987.$
27. $2.3\frac{1}{2}x + 5 = 240.$
28. $142x + 7 = 291.$
29. $62.4x + 50.4 = 300.$
30. $444x + 10 = 1342.$

13. Solving an Equation by Adding. If we have the equation

$$4x - 7 = 57,$$

we may add 7 to these equals. Then because $4x - 7 + 7$ is the same as $4x$, we have

$$4x = 64,$$

whence $x = 16$, by dividing by 4.

14. Solving an Equation by Multiplying. If we have the equation

$$\frac{2x}{7} = 12,$$

we may divide these equals by 2, and obtain

$$\frac{x}{7} = 6,$$

whence $x = 42$, by multiplying by 7.

Exercise 8. Equations

Solve by adding:

1. $x - 5 = 35.$
2. $4x - 9 = 31.$
3. $5x - 6 = 69.$
4. $6x - 3 = 93.$
5. $8x - 7 = 97.$
6. $12x - 6 = 150.$
7. $15x - 12 = 363.$
8. $2.7x - 0.8 = 15.4.$

Solve by multiplying:

9. $\frac{x}{9} = 15.$
10. $\frac{x}{12} = 11.$
11. $\frac{x}{27} = 22.$
12. $\frac{x}{3.5} = 4.8.$
13. $\frac{2x}{9} = 36.$
14. $\frac{4x}{7} = 64.$
15. $\frac{7x}{9} = 63.$
16. $\frac{2.4x}{5} = 16.8.$

15. Letters used in solving Problems. One of the chief uses of letters in algebra is in solving problems. A problem in arithmetic can often be more clearly solved in this way than by numbers alone.

For example, I am thinking of a number. When it is multiplied by 3, and 5 is added to the result, the sum is 38. What is the number?

Solution using a letter:

If I am thinking of n, then

$3n$ is 3 times the number,
$3n + 5$ is 5 added to $3n$,
and $3n + 5 = 38$, as stated in the problem.
Then $3n = 33$, by subtracting 5 from equals,
and $n = 11$, by dividing these equals by 3.

Check or *Proof.* $3 \times 11 + 5 = 38$.

Solution without using letters:

Because 3 times the number added to 5 equals 38, therefore if 5 is taken away from 38 there remains 3 times the number. But $38 - 5 = 33$. Therefore 33 is 3 times the number. Therefore once the number is $\frac{1}{3}$ of 33, or 11.

The solutions compared:

$3n + 5 = 38.$	38
Subtracting 5,	5
$3n = 33.$	$3\overline{)33}$
Therefore $n = 11.$	11

We therefore see that the two solutions are the same, but that the letters make the reasoning clearer.

Therefore in solving a problem by algebra,

(1) *Write a letter for the number sought.*
(2) *Use this letter in the statement of the problem.*
(3) *This will give an equation, as shown above.*
(4) *Solve this equation.*

Exercise 9. Problems

Represent the following algebraically:

1. 2 times some number.
2. The sum of 2 times some number and 7.
3. The sum of 4 times some number and 5.
4. The sum of 7 times some number and $2\frac{1}{2}$.
5. A number multiplied by 16, with 9 taken away.
6. The sum of some number and 10, divided by 7.
7. The sum of some number and 15, divided by 25.
8. The sum of 5 times some number and 6.
9. 3 times some number equals 21.
10. The sum of 2 times some number and 7 equals 17.
11. The sum of 5 times some number and 6 equals 36.
12. 5 times some number, less 21, equals 29.

Solve the following:

13. What number multiplied by 13 equals 247?
14. What number added to 39 equals 121?
15. What number diminished by $27\frac{3}{4}$ equals $15\frac{1}{8}$?
16. What number divided by 5 equals 17?
17. What number multiplied by $\frac{2}{3}$ equals 22?
18. What number added to $\frac{2}{3}$ equals $17\frac{1}{8}$?
19. What number diminished by $\frac{2}{3}$ equals $16\frac{2}{3}$?
20. What number divided by $\frac{2}{3}$ equals 12?
21. Twice what number is 1102?
22. A number is multiplied by 2, and 5 is taken from the product. The remainder is 55. What is the number?
23. A number is multiplied by 5, and 7 is added to the product. The sum is 82. What is the number?

24. The length of a schoolroom is 11 ft. more than the width. The length is 32 ft. What is the width?

25. A ball team has played 20 games. Out of these it has lost 9. How many has it won?

26. A ball team has lost 7 games and won twice this number, less 3. How many games has it played?

27. Our ball team played with the seventh-grade team. Our score was 3 less than twice theirs. Ours was 11. What was theirs?

28. Our ball team played with the Washington School team. Their score was 14 less than 3 times ours. Theirs was 7. What was ours?

29. A table is to be made such that the width is 6 in. more than half the length. The width is 48 in. What is the length?

30. A box is to be made that lacks 2 in. of being twice as long as it is wide. It is to be 17 in. long. How wide is it to be?

31. A steam hoist is so made that it can raise a weight that is twice the power used. What weight can be raised if the power is 1675 lb.?

32. In Ex. 31 what power is necessary to raise a weight of 2250 lb.?

33. There is a certain lever by which a man can lift 5 times his weight. If he can lift 794 lb., what is his weight?

34. There is a certain lever by which a man can lift 3.8 times his own weight. If he can lift 665 lb., what is his weight?

35. A man has a lever by which he can lift $4\frac{3}{5}$ times his own weight, lacking 7 lb. He can lift 683 lb. What is his weight?

16. Use of the Letter x. While we may represent a number by a small or large initial letter, as n for number, d for dollars, and F for feet, or by any other letters, *it is customary to represent by the letter x a number that is to be found.*

In the equations already solved, the number to be found has been represented both by x and by the initial letter. Hereafter, in general, x will be used, except where initials are more convenient in formulas.

Exercise 10. Equations

Find the value of x:

1. $9x = 315.$
2. $8x + 3 = 139.$
3. $7x + 9 = 142.$
4. $9x + 11 = 200.$
5. $11x + 13 = 134.$
6. $19x + 7 = 140.$
7. $31x + 12 = 260.$
8. $51x + 45 = 300.$
9. $73x - 11 = 354.$
10. $81x - 52 = 758.$

Make up problems for the following, and solve:

11. $2x + 8 = 30.$
12. $3x + 7 = 28.$
13. $5x + 12 = 72.$
14. $7x + 15 = 85.$
15. $9x + 23 = 212.$
16. $37x + 29 = 140.$
17. $4x - 17 = 23.$
18. $6x - 34 = 32.$
19. $8x - 51 = 357.$
20. $10x - 73 = 927.$
21. $13x - 69 = 100.$
22. $273x - 111 = 1800.$

23. If to twice a certain number I add 57, the result is 171. What is the number?

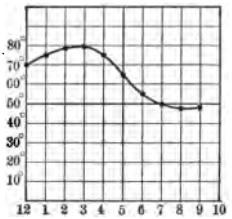
24. If from 5 times a certain number I take 57, the result is 228. What is the number?

25. If to 73 I add 6 times a certain number, the result is 181. What is the number?

CHAPTER III

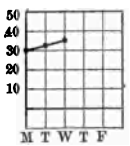
NEGATIVE NUMBERS

17. Curve Tracing. In this figure the successive hours of the day from noon to 9 P.M. are represented on a horizontal line, and the temperatures are represented by points on the vertical lines. It shows that at noon the temperature was 70°, at 1 P.M. 75°, at 2 P.M. 78°, at 3 P.M. 80°, and so on. If we connect these points by a curve, this line gives us a picture of the change in temperature for nine hours.

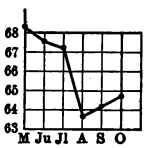


Exercise 11. Curve Tracing

1. The attendance in the eighth-grade class of a certain school was 30 on Monday, 32 on Tuesday, 35 on Wednesday, 30 on Thursday, and 28 on Friday. Part of the line (in this case made up of short straight lines) is here shown. Trace the entire line. Make a similar problem and trace the line.



2. The per cent of games won to games played up to the first day of May, June, and so on to December, by a certain team, was 68.4%, 67.6%, 67.2%, 63.7%, 64.2%, 64.7%, 65.5%, 67%. Trace the entire line of which a portion is here shown. Make a similar problem and trace the line.



Rule some paper and trace curves to show the following:

3. The temperature on a certain day varied as follows: 6 A.M. 50°, 7 A.M. 52°, 8 A.M. 55°, 9 A.M. 65°, 10 A.M. 68°, 11 A.M. 70°, noon 75°, 1 P.M. 76°, 2 P.M. 77°, 3 P.M. 77°, 4 P.M. 75°, 5 P.M. 73°, 6 P.M. 68°.

4. The pressure on the steam gauge of a boiler varies as follows: 8 A.M. 122 lb., 9 A.M. 120 lb., 10 A.M. 116 lb., 11 A.M. 120 lb., noon 114 lb., 12.30 P.M. 110 lb., 1 P.M. 115 lb., 2 P.M. 120 lb., 3 P.M. 116 lb., 4 P.M. 118 lb., 5 P.M. 114 lb.

5. A man's income for ten consecutive years, stated in hundreds of dollars, was as follows: 9.7, 10.1, 10.5, 12, 13.5, 15, 15.2, 15.7, 17, 20.

6. A boy's height from the age of 5 to the age of 15, stated in inches, varied as follows: 5 yr., 42; 6 yr., 44; 7 yr., 46; 8 yr., 49; 9 yr., 52; 10 yr., 54; 11 yr., 56; 12 yr., 58; 13 yr., 61; 14 yr., 63; 15 yr., 68.

7. A girl's height from the age of 5 to the age of 15, stated in inches, varied as follows: 5 yr., 42; 6 yr., 44; 7 yr., 45; 8 yr., 48; 9 yr., 50; 10 yr., 52; 11 yr., 54; 12 yr., 58; 13 yr., 60; 14 yr., 63; 15 yr., 65.

8. The population of the United States, in millions, for various years was as follows: 1820, 10; 1830, 13; 1840, 17; 1850, 23; 1860, 31; 1870, 39; 1880, 50; 1890, 63; 1900, 76; 1910, 92. In such a case it is convenient to use 0.1 in. to represent 10 million.

9. The population of the United States at a recent census, arranged according to age, in millions, was as follows: 10 years of age, 1.7; 20 years, 1.5; 30 years, 1.5; 40 years, 1.2; 50 years, 0.9; 60 years, 0.5; 70 years, 0.3; 80 years, 0.09; 90 years, 0.01. In such a case it is convenient to use 1 in. to represent 1 million, merely estimating the small fractions 0.09 and 0.01.

18. Numbers below Zero. Sometimes the temperature goes below zero. When it is necessary to distinguish between temperature below zero and temperature above zero, we write 10° above zero, + 10°; and 10° below zero, − 10°.

If the temperature is 20° above zero and it decreases 15°, it is then 5° above zero, or + 5°. If it decreases 5° more, it is then 0°. If it decreases 5° more, it is 5° below zero, or − 5°. If it decreases 20° more, it is 25° below zero, or − 25°.

We therefore find a new meaning for the signs + and −. They not only indicate addition and subtraction (signs of *operation*), but they tell on which side of zero a number is (signs of *quality*).

19. Positive Numbers. The numbers ordinarily used in arithmetic are called *positive numbers.*

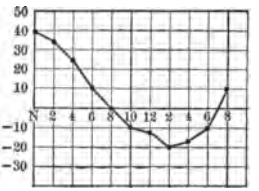
Thus 3°, 3 in., $\frac{3}{4}$, $\sqrt{3}$, are all positive numbers. If we wish to make this fact emphatic, we may write them thus: + 3°, + 3 in., + $\frac{3}{4}$, + $\sqrt{3}$, but otherwise the + sign is unnecessary here. The expression + 3 is read "positive 3" or "plus 3."

20. Negative Numbers. Numbers on the other side of zero from positive numbers are called *negative numbers.*

Thus − 3° is a negative number. If distance upwards, above the earth's surface, is called positive, distance below the surface may be called negative, so that we may have + 10 ft. and − 10 ft. The expression − 3 is read "negative 3" or "minus 3." We may think of zero as either positive or negative, since it divides the two classes of numbers.

If the temperature is 20° below zero, or − 20°, and it increases 5°, it is then 15° below zero, or − 15°. If it increases 10° more, it is 5° below zero, or − 5°. If it increases 5° more, it is then zero, or 0°. If it increases 5° more, it is 5° above zero, or + 5°.

21. Curve Tracing with Negative Numbers. If the temperature in Chicago, on a winter's day, varies from + 40° at noon to − 20° at 2 o'clock the next morning, as here shown, the curve tells us that it was below zero from 8 P.M. until about 7 the next morning.



We represent negative numbers below the horizontal line, which indicates zero.

Exercise 12. Curve Tracing

Trace the curves to show the following variations in temperature:

1. Noon 32°, 2 P.M. 40°, 4 P.M. 30°, 6 P.M. 28°, 8 P.M. 20°, 10 P.M. 5°, midnight −10°, 2 A.M. −12°, 4 A.M. −5°, 6 A.M. 0°, 8 A.M. 10°, 10 A.M. 30°, noon 35°.

2. Noon 45°, 2 P.M. 40°, 4 P.M. 38°, 6 P.M. 20°, 8 P.M. 0°, 10 P.M. −5°, midnight −6°, 2 A.M. −5°, 4 A.M. 0°, 6 A.M. 2°, 8 A.M. 2°, 10 A.M. 20°, noon 50°.

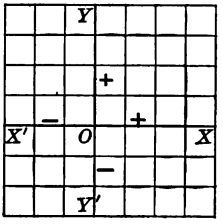
3. Noon 0°, 2 P.M. 5°, 4 P.M. 20°, 6 P.M. 15°, 8 P.M. 0°, 10 P.M. −2°, midnight −10°, 2 A.M. −8°, 4 A.M. −2°, 6 A.M. 10°, 8 A.M. 30°, 10 A.M. 32°, noon 40°.

4. Noon 10°, 1 P.M. 12°, 2 P.M. 14°, 3 P.M. 10°, 4 P.M. 0°, 5 P.M. −2°, 6 P.M. −5°, 7 P.M. −3°, 8 P.M. 0°, 9 P.M. −5°, 10 P.M. −12°, 11 P.M. −8°, midnight 0°.

5. Noon 5°, 1 P.M. 8°, 2 P.M. 10°, 3 P.M. 5°, 4 P.M. −5°, 5 P.M. −7°, 6 P.M. −8°, 7 P.M. −9°, 8 P.M. −12°, 9 P.M. −15°, 10 P.M. −20°, 11 P.M. −21°, midnight −20°.

6. Noon −2°, 1 P.M. 0°, 2 P.M. 5°, 3 P.M. 0°, 4 P.M. −5°, 5 P.M. −6°, 6 P.M. −9°, 7 P.M. −10°, 8 P.M. −15°, 9 P.M. −16°, 10 P.M. −20°, 11 P.M. −15°, midnight −10°.

22. Other Uses of Negative Numbers. If we call some special point on a line zero (0), we usually call distances to the right positive and distances to the left negative, just as we call distances up (as on the thermometer) positive and distances down negative. But because we usually call weight positive, we speak of the weight of a balloon (which pulls upward) negative.



These are some illustrations of negative numbers:

If a man is worth $1000, we may say that he has + $1000; but if he is $1000 in debt, we may say that he is worth − $1000.

If we call latitude north of the equator positive, we may call latitude south of the equator negative.

If we call longitude west of Greenwich positive, we may call longitude east of Greenwich negative.

If we call the motion of a piston rod of an engine positive when it is to the right, we should call the motion negative when it is to the left.

If we call steam pressure positive, we might speak of a vacuum as having negative pressure.

If we call distance above the earth's surface positive, we should call distance below the earth's surface negative.

We therefore see that negative numbers are just as real as positive numbers, for the temperature is just as real when the thermometer indicates that it is below zero as it is when the mercury rises above zero, and a man's debts are just as real as his capital.

In ancient times people used only whole numbers (integers). Other kinds of numbers were invented as they became necessary, and these are sometimes called artificial numbers. Artificial numbers like $\frac{3}{4}$, $\sqrt{3}$, and -3 all have their uses, as we have seen, not only in algebra but also in practical life.

23. Adding Negative Numbers. If we tie to a 10-pound weight a toy balloon that pulls upward 1 pound, what will the two together weigh?

To add a positive number to a negative number, take their numerical difference and prefix the sign of the numerically greater number.

Thus + 10 lb. and − 1 lb. are + 9 lb.;
+ 10 lb. and − 10 lb. are 0 lb.;
+ 10 lb. and − 15 lb. are − 5 lb.

That is, if a 10-pound weight is tied to a balloon that pulls upward 15 lb. (or weighs − 15 lb.), the two will weigh − 5 lb. We also see that − 10 lb. and − 5 lb. are − 15 lb.

Exercise 13. Addition

1. What is the combined weight of + 25 lb. and − 5 lb.? of + 25 lb. and − 25 lb.? of − 25 lb. and + 5 lb.?

2. What is the combined weight of 30 lb. and − 60 lb.? of 40 lb. and − 45 lb.? of 100 lb. and − 75 lb.? of − 100 lb. and − 75 lb.? of − 100 lb. and − 100 lb.?

3. A freight engine is switching in front of a station. If it runs 400 ft. to the right of the station (+ 400 ft.) and then backs 525 ft. (− 525 ft.), how many feet is it from the station? (Add 400 and − 525.)

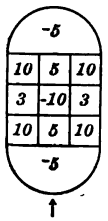
4. In drilling a well the drill is raised + 8 ft. above the surface of the ground. It is then dropped 15 ft. (−15 ft.). Where is it then with respect to the surface? (Add 8 and − 15. A negative distance above the surface means distance below the surface.)

5. A boy is fishing in deep water with a line 20 ft. long. If the tip of the pole is + 6 ft. above the water, how far is the sinker from the surface of the water, if it is 3 ft. from the hook? (Add 6 and − 20 and 3.)

6. A man who was $80 in debt paid $30. How much was he then in debt? Suppose he earns $50 more, how much is he then worth? (Add -80 and 30. Then add 50.)

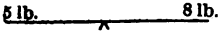
7. A man who was worth $3500 lost $1750 and then earned $900. How much was he then worth? (Add 3500, -1750, and 900.)

8. A man who was $350 in debt contracted another debt of $200. He then earned $1000. How much was he then worth?



9. A game is played by throwing bean bags in the direction of the arrow. Suppose the score stands -5, 3, 10, 10, 0, -10, 5, 10, 10, how much is the total score?

10. If this board without any weights at the ends just balances, and if I put 5 lb. at one end and 8 lb. at the other end, how much must I add to the 5 lb. to make it balance? Instead of adding to the 5 lb., how much must I add to the 8 lb. to make it balance?



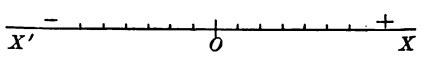
11. A boat that runs 16 mi. an hour in still water is going against a stream flowing 4 mi. an hour. What is the rate at which the boat will travel? (16 mi. and -4 mi. are how many miles?)

12. If a mine is opened 300 ft. above the base of a mountain and a shaft is sunk 700 ft., how much is the base of the shaft above or below the base of the mountain?

Add the following:

13.	14.	15.	16.	17.
75	-60	-30	-90	-35
-80	20	-40	-72	-75
10	30	70	-86	23

24. Subtracting Negative Numbers. If the temperature is $-10°$ at midnight and $+40°$ at noon, the difference in temperature is evidently 50°, for the mercury must rise 10° to reach 0°, and 40° more to reach $+40°$. Likewise, in this figure the difference between -2 and $+4$ is evidently 6; for a point must move 2 spaces to get from -2 to 0, and 4 more to reach $+4$.



To subtract a negative number we may obtain the same result by adding a positive number with the same numerical value.

That is, $4-(-2)=4+(+2)=6$. Likewise, $-4-(-2)=-4+2=-2$. We also see that $-4-2=-6$, as in § 23.

Exercise 14. Subtraction

1. The temperature on one January morning in Denver was $+9°$, and the next day it was $-5°$. What was the difference in temperature?

2. If there is a house for every number, how many houses would you pass in going from 42 East Washington Street to 15 West Washington Street, including both these houses?

3. Jefferson Street is 5 blocks east of Adams Street, and Monroe Street is 8 blocks west of Adams Street. Monroe Street is how many blocks west of Jefferson Street?

Subtract the following:

4. -3 from 0.
5. -5 from 12.
6. -9 from 17.
7. -7 from 15.
8. -8 from 21.
9. -6 from 30.
10. -6 from -9.
11. -4 from -8.
12. -39.4 from 82.7.
13. $-48\frac{2}{3}$ from $15\frac{1}{3}$.
14. -2.75 from 2.75.
15. -2.7 from -2.7.

25. Multiplying and Dividing Negative Numbers. We multiply and divide negative numbers just as we multiply and divide positive numbers. If a man has $-\$5$ (is \$5 in debt), he will have $-\$10$ if he is twice as much in debt.

That is
$$2 \times (-\$5) = -\$10,$$
$$(-\$10) \div 2 = -\$5.$$

Exercise 15. Multiplication and Division

1. If one balloon pulls up 300 lb. (weighs -300 lb.), what will be the upward pull of 3 such balloons? Represent the result as a negative number.

2. If the thermometer indicates $-4°$, what is the temperature when it indicates half as much below zero?

3. If a carrier pigeon can fly 40 mi. an hour in still air, at what rate will it fly against a 20-mile wind? against a wind that blows twice as fast? three times as fast?

4. A checker of bales of cotton allows 500 lb. to the bale. He finds one bale 18 lb. short, a second bale twice as much short, and a third bale half as much short in weight. Express these shortages in algebraic language.

5. A man's debts amounted last year to \$375. The year before they were 4 times as much. This year he has paid his debts and has \$925 in the bank. What is the difference between his financial standing two years ago and now?

Multiply the following:

6.	7.	8.	9.	10.	11.
-25	-47	-39	-57	-63	-77
3	6	8	21	18	33

Divide the following:

12. $-125 \div 25$. **13.** $-121 \div 11$. **14.** $-49.3 \div 1.7$.

26. Multiplying and Dividing by Negative Numbers. Because $2 \times (-5) = -10$, we see, by interchanging multiplier and multiplicand, that $(-5) \times 2 = -10$.

That is,
$$2 \times (-5) = -10,$$
$$(-5) \times 2 = -10.$$

We therefore conclude that
$$(-5) \times (-2) = -(-10) \text{ or } +10.$$

If two numbers have like signs, their product is positive; if they have unlike signs, their product is negative.

That is,
$$(+a) \times (+b) = +ab,$$
$$(-a) \times (-b) = +ab,$$
$$(+a) \times (-b) = -ab,$$
$$(-a) \times (+b) = -ab.$$

Dividing these four products by the divisors, we have
$$(+ab) \div (+a) = +b,$$
$$(+ab) \div (-a) = -b,$$
$$(-ab) \div (+a) = -b,$$
$$(-ab) \div (-a) = +b.$$

We therefore see that, in division,

If two numbers have like signs, their quotient is positive; if they have unlike signs, their quotient is negative.

Exercise 16. Multiplication and Division

1. 21×19.
2. $21 \times (-19)$.
3. $(-21) \times 19$.
4. $(-21) \times (-19)$.
5. $37 \times (-12)$.
6. $(-17) \times (-14)$.
7. $(-32) \times 48$.
8. $625 \div 25$.
9. $(-625) \div 25$.
10. $625 \div (-25)$.
11. $(-625) \div (-25)$.
12. $288 \div (-24)$.
13. $(-432) \div 36$.
14. $(-576) \div (-48)$.

CHAPTER IV

THE FUNDAMENTAL OPERATIONS

27. Factors. Quantities which are multiplied together are called the *factors* of the product.

28. Square. If two factors are equal, their product is called a *square.*

Thus we have already seen that $a^2 = aa$, and $2^2 = 4$.

29. Power. The product arising from taking a quantity a certain number of times as a factor is called a *power.*

Thus $2 \times 2 \times 2 = 2^3$, or 8. Likewise a^4 is the fourth power of a, and equals $aaaa$. Hence $a^4 \div a = a^3$.

30. Exponent. In the quantity a^4, 4 is called the *exponent* of a, and indicates the power to which a is raised.

In a quantity like a or x, the exponent 1 is understood.

31. Coefficient. A numerical factor written before a letter is called the *coefficient* of the letter.

Thus 2 is the coefficient of a in the quantity $2a$. In a quantity like x, a, or m^2, the coefficient 1 is understood.

32. Algebraic Expression. A letter, or a collection of letters, or of letters and numbers representing operations ($+$, $-$, $\times$, $\div$, etc.), is called an *algebraic expression.*

33. Term. An algebraic expression containing neither the $+$ nor the $-$ sign of operation is called a *term.*

The terms of $2a^2 - 3ab$ are $2a^2$ and $3ab$. The *literal factor* of the term $3ab$ is ab and the *numerical factor* is 3.

34. Algebraic Expressions classified according to Terms. An algebraic expression of one term is called a *monomial;* of two terms, a *binomial;* of three terms, a *trinomial;* of several terms, a *polynomial.*

Thus $4a^2b$ is a monomial; $2 + 3ax$ is a binomial.

Exercise 17. Terms Employed

Write the factors of:

1. $2\pi r$. 2. πd. 3. lh. 4. $2a^2bc$.

Write the coefficients of the literal part in the following:

5. x^2. 6. $3x^4$. 7. $5abc$. 8. $\frac{1}{2}xyz$.

Write the exponents of r in the following:

9. πr^2. 10. $2\pi r$. 11. $\frac{4}{3}\pi r^3$. 12. $\pi r^2 h$.

Write the squares of the following:

13. x. 14. $4x$. 15. -7. 16. $-9m$.

Write the third powers of the following:

17. m. 18. $3x$. 19. -2. 20. $-5m$.

21. Is $2xy$ a monomial, a binomial, or a trinomial?

22. What kind of an expression, as to number of terms, is $2 + x + y$?

23. Select the trinomials from the following: $a + 3b$; $a^2 - b + 4$; $x^2 - y^2$; $x^4 + x^2y^2 + y^4$; $x^5 + 3x^4 + 2x^3 + 5$.

24. In the expression $5a^3$, which number is a coefficient and which is an exponent?

25. In $7x$, what exponent of x is understood? In x^4, what coefficient of x^4 is understood?

26. In the monomial a, what coefficient and what exponent are understood?

35. Addition. We add quantities in algebra in much the same way that we add denominate numbers.

For example, study these sums:

2 ft.	$\frac{2}{15}$	$2f$	$2 \cdot 5$	$2\,abc$	$3x$
3 ft.	$\frac{3}{15}$	$3f$	$3 \cdot 5$	$3\,abc$	$-2x$
7 ft.	$\frac{7}{15}$	$7f$	$7 \cdot 5$	$7\,abc$	x
12 ft.	$\frac{12}{15}$	$12f$	$12 \cdot 5$	$12\,abc$	$2x$

In each case we add the coefficients, writing the common factor after this sum.

Also study the following:

2 ft. 6 in.	$2f + 6i$	$2x + 6y$
8 ft. 2 in.	$8f + 2i$	$8x - 2y$
10 ft. 8 in.	$10f + 8i$	$10x + 4y$

Exercise 18. Addition

Add the following:

1.	5.	9.
3 ft. 2 in.	$2a + 3b + 4c$	$x + y + z$
5 ft. 3 in.	$5a + 2b + 6c$	$x + y - z$

2.	6.	10.
$3f + 2i$	$4a + 3b + 9c$	$2x + 3y + 5z$
$5f + 3i$	$2a + 2b - 3c$	$3x + 7y + 8z$

3.	7.	11.
$3 \cdot 6 + 2 \cdot 4$	$7a + 2b + c$	$5x + 2y + z$
$5 \cdot 6 + 3 \cdot 4$	$3a + 8b - 9c$	$7x + 3y + z$

4.	8.	12.
$7x + 8y$	$14a + 18b + 10c$	$2x + 8y - 7z$
$9x + 3y$	$3a - b + 7c$	$5x - y + 6z$

36. Unknown Quantity. A letter in an equation, for which a value is to be found, is called an *unknown quantity.*

Thus in the equation $3x + 19 = 49$, the unknown quantity is x.

37. Root. The value of an unknown quantity in an equation is called the *root* of the equation.

In the equation $3x + 19 = 49$, the root is 10.

38. Members of an Equation. The two sides of an equation, separated by the sign of equality, are called the *members* of the equation.

The one to the left is the *first member;* the one to the right is the *second member.* The members are often called the *sides* of the equation.

39. Transpose. In the equation $2x = x + 7$ we may subtract x from both members and we have $x = 7$. We then say that we have *transposed* x.

Likewise if we have $7x - 3 = 46$, we may *add* 3 to both members and we have $7x = 49$. We then say that we have *transposed* -3.

We therefore see that *we may transpose a term from one member of an equation to the other by changing its sign.*

40. Degree. The number of literal factors in a term is called its *degree.*

Thus $4a^2$ and $\frac{1}{2}ab$ are both of the second degree, for $4a^2$ contains the two literal factors a and a, and $\frac{1}{2}ab$ contains the two literal factors a and b.

A term like x, having the exponent 1 understood, so that $x = x^1$, is said to be of the *first degree.*

The term $5ax$ is of the second degree, considering both a and x, but we also speak of it as of the *first degree in* x.

41. Simple Equation. An equation of which the terms are all of the first degree in the unknown quantity is called *a simple equation.*

A simple equation is also called an *equation of the first degree.*

42. Equations involving Addition. If we have the equation $3x + 8x = 132$, we proceed as follows:

$$3x + 8x = 132.$$
$$11x = 132, \text{ by adding.}$$
$$x = 12, \text{ by dividing by } 11.$$

Exercise 19. Equations

Solve the following:

1. $2x + 3x = 75.$
2. $3x + 4x = 84.$
3. $3x + 5x = 96.$
4. $7x + 4x = 143.$
5. $8x + 6x = 154.$
6. $7x + 9x = 176.$
7. $9x + 4x = 169.$
8. $10x + 3x = 117.$
9. $12x + 5x = 306.$
10. $15x + 7x = 462.$
11. $x + x = 24.$
12. $x + 2x = 72.$
13. $x + 9x = 300.$
14. $x + 13x = 308.$
15. $x + \frac{1}{2}x = 165.$
16. $x + \frac{1}{4}x = 165.$
17. $x + \frac{3}{4}x = 154.$
18. $x + 0.5x = 195.$
19. $2x + 2\frac{1}{2}x = 165.$
20. $3x + 5\frac{1}{8}x = 275.$

21. If to twice a certain number I add 7 times that number, the result is 261. What is the number?

22. If to 3 times a certain weight I add 15 times that weight, the result is 720 lb. What is the weight?

23. If to $2\frac{1}{2}$ times a certain weight I add $3\frac{1}{8}$ times that weight, the result is 225 lb. What is the weight?

24. If to 7 times a certain number I add 6 times the number, and then add 5 times the number, the result is 126. What is the number?

25. If to 15 times a certain quantity of milk a dealer adds 6 times that quantity, he has 147 gallons. What is the quantity?

43. Equations involving Per Cents. The equation is very useful in percentage. For example: After adding 15% to the cost of a horse a dealer sold it for $184. What did the horse cost the dealer?

Let x represent the *number* of dollars in the cost.

Then $x + 0.15x =$ the selling price.

But $184 = the selling price.

Therefore $x + 0.15x = 184$,

or, by adding, $1.15x = 184$.

Therefore $x = 160$, by dividing by 1.15.

Therefore the horse cost $160. *Ans.*

We may check this result by adding 15% of $160 to $160, and we have $160 + $24 = $184, the selling price.

Exercise 20. Per Cents

1. What number increased by 10% of itself equals 176?

2. What number increased by 25% of itself equals 200?

3. What number increased by $33\frac{1}{3}$% of itself equals 200?

4. What number increased by $12\frac{1}{2}$% of itself equals 189?

5. What number increased by $66\frac{2}{3}$% of itself equals 275?

6. A boy weighs 60 lb. now, which is 20% more than he weighed four years ago. What did he weigh then?

7. A certain sum increased by 8% of itself is $135. What is the sum?

8. A certain sum plus 6% interest for one year amounts to $265. What is the sum?

9. A certain school has 10% more pupils than it had last year. It now has 132 pupils. How many pupils did it have last year?

10. A merchant saved $2497.50 last year, which was 11% more than he saved the year before. How much did he save the year before?

Solve the following:

11. $1.25x = 13.75$.

12. $1.40x = 26.60$.

13. $1.75x = 29.75$.

14. $1.12\frac{1}{2}x = 540$.

15. $1.16\frac{2}{3}x = 777$.

16. $x + 0.25x = 13.75$.

17. $x + 0.40x = 2100$.

18. $x + 0.75x = 8470$.

19. $x + 12\frac{1}{2}\%\,x = 720$.

20. $x + 33\frac{1}{3}\%\,x = 164$.

21. A dealer sold some land at 5% above cost. He sold it for $3412.50. What did it cost him?

22. A dealer sold some furniture at 15% above cost. He sold it for $276. What did it cost him?

23. A man's expenses in one year were increased 17% the next year, when they were $1872. What were they the first year?

24. The steam pressure on a boiler was 126 lb. per square inch at 10 A.M., which was 5% more than it was at 9 A.M. What was the pressure at 9 A.M.?

25. A flywheel of an engine is making 91 revolutions a minute, an increase of $8\frac{1}{3}$% over its speed an hour ago. What was its speed an hour ago?

26. A poultry raiser has 371 hens, which is 6% more than he had last month. How many did he have last month?

27. A year's interest on a certain sum at 6% is $45. What is the sum?

28. The population of a village is now 4950, an increase of $12\frac{1}{2}$% over the population ten years ago. What was it ten years ago?

29. The number of pupils in a school is 147, an increase of 5% on the number last year. What was it last year?

30. A man sold a lot of Texas ponies for $5664, thus making a profit of 18%. What did the ponies cost him?

44. Subtraction. We subtract quantities in algebra in much the same way that we subtract denominate numbers. For example, study these cases of subtraction:

21 ft.	$\frac{21}{25}$	$21\,a$	$21\,ax$	$21 \cdot 3$
4 ft.	$\frac{4}{25}$	$4\,a$	$4\,ax$	$4 \cdot 3$
17 ft.	$\frac{17}{25}$	$17\,a$	$17\,ax$	$17 \cdot 3$

We have found (§ 24) that to subtract a negative number we may change the sign and add. Thus:

4	$4\,x$	$4\,ax$	$5\,abc$	$-3\,abx$
-2	$-2\,x$	$-2\,ax$	$-abc$	$-7\,abx$
6	$6\,x$	$6\,ax$	$6\,abc$	$4\,abx$

We may also have a negative remainder, as in the subtraction of $10\,x$ from $3\,x$.

$$\begin{array}{r} 3\,x \\ 10\,x \\ \hline -7\,x \end{array}$$

If the temperature drops 10° from 3°, the thermometer registers $-7°$.

Exercise 21. Subtraction

1.	**5.**	**9.**
13 ft. 7 in.	$6\,ax$	$2\,x + 3\,y$
6 ft. 3 in.	$4\,ax$	$x - 2\,y$
2.	**6.**	**10.**
$13\,x + 7\,y$	$6\,ax$	$4\,x + 7\,y$
$6\,x + 3\,y$	$-4\,ax$	$5\,x - 2\,y$
3.	**7.**	**11.**
$15\,x + 9\,y$	$9\,axy$	$5\,xy + 3\,z$
$7\,x + y$	$10\,axy$	$2\,xy - 3\,z$
4.	**8.**	**12.**
$27 \cdot 2 + 16 \cdot 3$	$-12\,abc$	$17\,xy - 12\,z$
$12 \cdot 2 + 18 \cdot 3$	$-4\,abc$	$20\,xy - 16\,z$

45. Equations involving Subtraction. If we have the equation $7x - 2x = 235$, we proceed as follows:

$$7x - 2x = 235.$$
$$5x = 235, \text{ by subtracting.}$$
$$x = 47, \text{ by dividing by } 5.$$

Exercise 22. Equations

Solve the following:

1. $9x - x = 72.$
2. $8x - 2x = 354.$
3. $7x - 5x = 238.$
4. $13x - 5x = 184.$
5. $17x - 6x = 242.$
6. $19x - 5x = 294.$
7. $23x - 7x = 336.$
8. $25x - 9x = 672.$
9. $27x - 8x = 228.$
10. $31x - 4x = 324.$
11. $2x - x = 3\frac{1}{2}.$
12. $2x - \frac{1}{2}x = 63.$
13. $3x - \frac{1}{4}x = 88.$
14. $3x - \frac{3}{4}x = 81.$
15. $5x - 1\frac{1}{2}x = 84.$
16. $7x - 3\frac{1}{4}x = 150.$
17. $2\frac{1}{2}x - \frac{1}{4}x = -126.$
18. $3\frac{1}{4}x - \frac{3}{4}x = -120.$
19. $2.7x - 1.3x = 29.4.$
20. $3.9x - 2.8x = 10.01.$

21. If from 9 times a certain number I take 6 times the number, the result is 321. What is the number?

22. If from 16 times a certain weight I take 9 times the weight, the result is 161. What is the weight?

23. If from 17 times a certain number I take 8 times the number, the result is 333. What is the number?

24. If from 23 times a certain quantity of grain there is taken 15 times the quantity, the result is 248 bu. What is the quantity?

25. If from 26 times a certain quantity of oil there is taken 17 times that quantity, there remains 603 gal. What is the quantity?

46. Equations involving Per Cents. A man sold a house for \$4200, thereby losing $12\frac{1}{2}\%$ on the cost. What did it cost him?

Let x represent the *number* of dollars in the cost.

Then $x - 0.12\frac{1}{2}x =$ the selling price.

But \$4200 = the selling price.

Therefore $x - 0.12\frac{1}{2}x = 4200$.

Subtracting, $0.87\frac{1}{2}x = 4200$.

Dividing by $0.87\frac{1}{2}$, or by $\frac{7}{8}$, $x = 4800$.

Therefore the house cost him \$4800. *Ans.*

Check. \$4800 − $12\frac{1}{2}\%$ of \$4800 = \$4800 − \$600 = \$4200.

Exercise 23. Per Cents

1. What number decreased by 8% of itself equals 184?

2. What number decreased by 25% of itself equals 360?

3. What number decreased by $33\frac{1}{3}\%$ of itself equals 160?

4. A piece of cloth after shrinking $1\frac{1}{2}\%$ of its length is 157.6 yd. long. How long was it before shrinking?

5. A stock of goods after being damaged $66\frac{2}{3}\%$ by fire is worth \$1600. What was it worth before the fire?

6. From a barrel $12\frac{1}{2}\%$ of the vinegar leaked out, leaving 28 gallons. How many gallons of vinegar were there at first?

7. An agent collected some money for a man, and after deducting his commission of 5% he remitted \$522.50. How much did he collect?

8. An agent collected some money for a man, and after deducting his commission of $2\frac{1}{2}\%$ he remitted \$1072.50. How much did he collect?

9. From an oil tank 17% of the oil was drawn off, leaving 10,541 gallons. How many gallons of oil were there in the tank at first?

10. Find a number such that 10% of it less 7 equals 19.

11. What sum increased by 1% of itself amounts to $2585.60?

12. Find a number which increased by 17% of itself, and then decreased by 36, equals 198.

13. A dealer sold some goods for $2566.50, thus losing 13%. What did the goods cost him?

14. A man lost 30% of his library by fire. He had 630 books left. How many had he before the fire?

15. A collection agency charges 4% for its services in collecting a debt, and remits $912. How much did it collect?

16. The sum of a certain number, $33\frac{1}{3}$% of the number, and 25% of the number, less 7, equals $4\frac{1}{12}$. What is the number?

17. A bank charges 0.1% exchange on a draft. The entire cost of draft and exchange is $1851.85. What is the face of the draft?

18. An agent charges 5% for collecting rents for Mr. Glover. He deducts his commission and remits $332.50. How much did he collect?

19. Half of the remainder found by subtracting 7 from a certain number equals 25% of the number. What is the number?

20. An agent bought a building lot for Mr. Roberts, charging him 3% commission. Mr. Roberts sent him the price of the lot and commission, amounting to $2626.50. What did the lot cost?

21. The income of a certain store increased $16\frac{2}{3}$% the second year it was open, and the income the third year was 25% more than the second year. The income being $3500 the third year, what was it the first year?

22. At what rate will \$320 produce \$19.20 interest in two years?

If the rate for 1 yr. is $r\%$, for 2 yr. it is $2r\%$.
Since $2r\%$ of \$320 = \$19.20,

$$r\% = \tfrac{1}{2} \text{ of } \frac{\$19.20}{\$320} = 3\%.$$

23. What per cent of \$75 is \$3.75?

24. What per cent of \$15.50 is \$0.62?

25. If $x\%$ of \$156 is \$4.68, find the value of x.

26. If $x\%$ of \$730 is \$65.70, find the value of x.

27. What sum increased by 6% of itself equals \$1007?

28. On what sum is \$11.25 the interest for 1 year at $4\frac{1}{2}\%$? at 5%? at 3%? at 2%?

29. At what rate of interest will \$450 produce \$15.75 in 1 year? in 2 years? in 9 months?

30. At what rate will \$240 produce \$28.80 interest in 2 years? in 3 years? in 4 years?

31. How long will it take \$350 and interest to amount to \$386.75 at $3\frac{1}{2}\%$? at 5%? at 4%?

32. At what rate will \$260 and interest amount to \$282.75 in $3\frac{1}{2}$ years? in 5 years? in $2\frac{1}{2}$ years?

33. What sum will amount, with interest, to \$381.50 in 2 years at $4\frac{1}{2}\%$? at 5%? at 6%? at 2%?

34. A man gained 15% on his capital and then had \$8625. How much had he at first?

35. A man has 27% of his capital invested in a farm. The farm is worth \$1917. How much is his capital?

36. What sum increased by 3% of itself amounts to \$785.89?

37. What sum decreased by 7% of itself is reduced to \$709.59?

38. An agent charges $5\frac{1}{2}\%$ for collecting rents for Mr. Roberts. He deducts his commission and remits $340.20. How much did he collect?

39. The cost of a draft, less the discount at 0.1%, is $9740.25. What is the face?

40. The cost of a draft, including the premium at 0.1%, is $10,760.75. What is the face?

41. A man pays an agent $7725 for buying a house, which includes the agent's commission of $225. What was the per cent of commission?

42. A man pays an agent $5610 for buying a house, which includes the agent's commission of 2%. What did the agent pay for the house?

43. A man bought two horses at the same price each. He sold the two for $198.90, gaining 11% on one and 10% on the other. What did he pay for them?

44. A man bought two horses at the same price each. He sold the two for $163.20, gaining 10% on one and losing 6% on the other. What did he pay for them?

45. A man increases his original capital by 7%, and the next year he decreases what he then had by 10%. He then has $8667. What was his original capital?

46. A mine increased its income 11% in one year, and the next year it increased this new income 10%. The income then amounted to $335,775 a year. What was its income at first?

47. A man lends $250 for a year at a certain per cent, and the next year he lends the $250 and the first year's interest at the same rate. The sum of principal and interest at the end of the first year is $260. What is the rate? What is the sum at the end of the second year?

47. Multiplication. We multiply quantities in algebra in much the same way that we multiply denominate numbers. For example, study these cases of multiplication:

3 ft. 2 in.	$3f + 2i$	$3x + 2y$
4	4	4
12 ft. 8 in.	$12f + 8i$	$12x + 8y$

The multiplier may also contain a letter, thus:

$3x + 2y$	$3x + 2y$	$3x - 2y$
a	$4a$	$-4a$
$3ax + 2ay$	$12ax + 8ay$	$-12ax + 8ay$

We indicate the multiplication of $a + b$ by x thus: $x(a + b)$.

Exercise 24. Multiplication

Multiply:

1.	**6.**	**11.**
$3a + 2b$	$52x + 37y$	$x + y + z$
5	$2a$	-3
2.	**7.**	**12.**
$5a - 3b$	$49x - 56y$	$x - 2y + z$
6	12	$-4a$
3.	**8.**	**13.**
$27x + 15y$	$59x + 47y$	$3x - 4y - 5z$
7	$23a$	$-15ab$
4.	**9.**	**14.**
$26p + 12q$	$76x - 37y$	$xy + yz + 1$
8	$31a$	$-a$
5.	**10.**	**15.**
$37m + 15n$	$37x - 96y$	$ab + cd + ef$
12	$23m$	$-x$

48. Multiplication involving Powers. If two factors have the same letter, we shall have in the product a power greater than the first power.

For example, study these cases of multiplication:

$$\begin{array}{r} a \\ \underline{a} \\ a^2 \end{array} \qquad \begin{array}{r} ab \\ \underline{a} \\ a^2b \end{array} \qquad \begin{array}{r} -a^2 \\ \underline{a} \\ -a^3 \end{array} \qquad \begin{array}{r} -a^2b \\ \underline{a} \\ -a^3b \end{array} \qquad \begin{array}{r} \pi r^2 \\ \underline{r} \\ \pi r^3 \end{array} \qquad \begin{array}{r} 4\,\pi r^2 \\ \underline{\frac{1}{3}\,r} \\ \frac{4}{3}\,\pi r^3 \end{array}$$

$$\begin{array}{r} a+b \\ \underline{a} \\ a^2+ab \end{array} \qquad \begin{array}{r} a^2+2\,ab \\ \underline{ab} \\ a^3b+2\,a^2b^2 \end{array} \qquad \begin{array}{r} a^2-2\,ab \\ \underline{-3\,ab} \\ -3\,a^3b+6\,a^2b^2 \end{array}$$

Exercise 25. Multiplication involving Powers

Multiply:

1.
$$\begin{array}{r} a^2 \\ \underline{a^2} \end{array}$$

2.
$$\begin{array}{r} a^2x \\ \underline{a^2} \end{array}$$

3.
$$\begin{array}{r} 3\,a^3x \\ \underline{2\,a^2} \end{array}$$

4.
$$\begin{array}{r} 5\,\pi r^2 \\ \underline{2\,r} \end{array}$$

5.
$$\begin{array}{r} 27\,a^2bc \\ \underline{12\,ab} \end{array}$$

6.
$$\begin{array}{r} a+b \\ \underline{b} \end{array}$$

7.
$$\begin{array}{r} a^2+b^2 \\ \underline{a^2} \end{array}$$

8.
$$\begin{array}{r} a^2-b^2 \\ \underline{-2\,a} \end{array}$$

9.
$$\begin{array}{r} a^2-3\,b^2 \\ \underline{4\,a^2} \end{array}$$

10.
$$\begin{array}{r} a^3-3\,b^3 \\ \underline{-7\,a^2} \end{array}$$

11.
$$\begin{array}{r} a+b+c \\ \underline{a} \end{array}$$

12.
$$\begin{array}{r} a^2+2\,a+1 \\ \underline{a^2} \end{array}$$

13.
$$\begin{array}{r} x^2-2\,xy+y^2 \\ \underline{-xy} \end{array}$$

14.
$$\begin{array}{r} 3\,x^2-4\,xy-7\,y^2 \\ \underline{-2\,xy} \end{array}$$

15.
$$\begin{array}{r} 7\,x^2-5\,xy-3\,y^2 \\ \underline{9\,xy} \end{array}$$

49. Multiplying by a Binomial. In multiplying by 25 we first multiply by 5 and then by 2 (tens), and add the two products. We may, if we wish, first multiply by 2 (tens) and then by 5. Similarly, to multiply by $a + b$ we first multiply by a and then by b, and add the products. Study the following:

$$\begin{array}{r} 143 \\ 25 \\ \hline 715 \\ 286 \\ \hline 3575 \end{array} \qquad \begin{array}{r} a - 3b \\ a + b \\ \hline a^2 - 3ab \\ ab - 3b^2 \\ \hline a^2 - 2ab - 3b^2 \end{array} \qquad \begin{array}{r} x + y \\ x - y \\ \hline x^2 + xy \\ -xy - y^2 \\ \hline x^2 \qquad - y^2 \end{array}$$

Exercise 26. Binomial Multipliers

Multiply:

1. $a + 3b$ by $a + b$.
2. $2a + 3b$ by $a + b$.
3. $3a - 7b$ by $a + b$.
4. $a + 2b$ by $2a - b$.
5. $3a - 2b$ by $4a + b$.
6. $2x + 3y$ by $x + 2y$.
7. $5x - 7y$ by $5x - 3y$.
8. $ab + 3$ by $2ab + 5$.
9. $3xy + 4$ by $5xy - 2$.
10. $x^2y^2 + 1$ by $x^2y^2 + 1$.
11. $a^2 + b^2$ by $a + b$.
12. $a^2 + b^2$ by $a - b$.
13. $a - b$ by $a + b$.
14. $x - y$ by $x + y$.
15. $x^2 - y^2$ by $x^2 + y^2$.
16. $2x - y$ by $2x + y$.
17. $3x^2 - 1$ by $3x^2 + 1$.
18. $5xy - 7$ by $5xy + 7$.
19. $4x^2y^2 - 1$ by $4x^2y^2 + 1$.
20. $7xyz - 2$ by $7xyz + 2$.

21. If the difference of two quantities is multiplied by their sum, how is the product related to the quantities?

22. By Ex. 21 multiply $a - 1$ by $a + 1$; $20 - 1$ by $20 + 1$; 19 by 21; 29 by 31; 59 by 61; $a - 2$ by $a + 2$.

23. Multiply $a^2 + 2ab + b^2$ by $a + b$.

24. Multiply $a + b$ by $a + b$, and the product by $a + b$.

50. Division of a Monomial. Since division is the inverse of multiplication, it follows from $a \cdot ab = a^2b$,

that $\qquad a^2b \div a = ab, \quad$ and $\quad a^2b \div ab = a;$

from $\qquad a \cdot (-a^2b) = -a^3b,$

that $\quad (-a^3b) \div a = -a^2b, \quad$ and $\quad (-a^3b) \div (-a^2b) = a;$

and from $\qquad (-a) \cdot (-ab^2) = a^2b^2,$

that $\quad a^2b^2 \div (-a) = -ab^2, \quad$ and $\quad a^2b^2 \div (-ab^2) = -a.$

If two quantities have like signs, their quotient is positive; if they have unlike signs, their quotient is negative.

In dividing a quantity affected by an exponent by the same quantity affected by a smaller exponent, the quotient is the quantity affected by the difference of the exponents.

That is, $a^7 \div a^3 = a^4$, $a^2b^5 \div ab^2 = ab^3$. This is evident from the fact that

$$\frac{a^7}{a^3} = \frac{aaaa\cancel{a}\cancel{a}\cancel{a}}{\cancel{a}\cancel{a}\cancel{a}} = a^4.$$

Exercise 27. Division

1. $a^2 \div a.$
2. $-a^2 \div a.$
3. $a^2 \div -a.$
4. $-a^2 \div -a.$
5. $x^4 \div x^2.$
6. $x^4 \div -x^3.$
7. $a^2x^7 \div -a^2.$
8. $-x^7 \div -x^4.$
9. $-10x^9 \div 5x^2.$
10. $-5x^9 \div x^6.$
11. $-15x^9 \div -3x^4.$
12. $-25a^2x^5 \div -5ax^4.$
13. $125a^4b^3 \div 25a^4b.$
14. $144x^5y^6 \div 12x^5y^4.$
15. $150a^2b^2c^2 \div 25abc.$
16. $-175a^2b^3c^4 \div 25a^2.$
17. $-169x^2y^4 \div -13xy^4.$
18. $-625a^2b^2c^5 \div 125a^2bc.$
19. $1728a^2b^3 \div 144ab.$
20. $1728a^2b^3 \div -12a.$
21. $-1560a^3b^4 \div -24a^3b.$
22. $-1680a^4b^5 \div 48a^4b^5.$
23. $-62.5m^7n^7 \div -1.25m^5.$
24. $-1.69a^9b \div -1.3a^7b.$

51. Division. We divide polynomials in much the same way that we divide compound numbers. For example, study these cases of division:

$$2\overline{)14 \text{ ft. } 8 \text{ in.}} \qquad x\overline{)14x^2 + 8xy} \qquad 2x\overline{)14x^2 + 8xy}$$
$$7 \text{ ft. } 4 \text{ in.} \qquad 14x + 8y \qquad 7x + 4y$$

$$2\overline{)14f + 8i} \qquad 2\overline{)14x + 8y} \qquad -2x\overline{)14x^2 - 8xy}$$
$$7f + 4i \qquad 7x + 4y \qquad -7x + 4y$$

Remember that $(+ab) \div (-a) = -b$, and $(-ab) \div (-a) = +b$, by § 26.

Exercise 28. Polynomials

1.	**9.**	**17.**
$3\overline{)27 \text{ ft. } 9 \text{ in.}}$	$3\overline{)15a - 27b}$	$x\overline{)2x+3xy+xz}$
2.	**10.**	**18.**
$3\overline{)27x + 9y}$	$3a\overline{)15a^2 - 27ab}$	$-x\overline{)3x-4xy-xz}$
3.	**11.**	**19.**
$3x\overline{)27x^2+9xy}$	$25a\overline{)25a^2 + 125a}$	$-x\overline{)5x^2-7xy+xz}$
4.	**12.**	**20.**
$3x\overline{)27xy+9xz}$	$19a\overline{)57ab - 38a^2}$	$x^2\overline{)7x^2+8x^3+9x^4}$
5.	**13.**	**21.**
$3xy\overline{)27x^2y-9xy}$	$3x^2\overline{)84x^2-27x^2y}$	$2x\overline{)2x^2+4x^3+6x^4}$
6.	**14.**	**22.**
$3x^2\overline{)27x^2-9x^2y}$	$4xy\overline{)64x^2y^2-28xy}$	$-x\overline{)6x^2+9xy-12x}$
7.	**15.**	**23.**
$3y^2\overline{)27x^2y^2-9y^2}$	$7x^2\overline{)84x^2y^2-133x^2}$	$5x^2\overline{)5x^4+10x^3-5x^2}$
8.	**16.**	**24.**
$3\overline{)27xyz-9z^2}$	$3abc\overline{)111a^2bc-9ab^2c}$	$7x^3\overline{)14x^6-7x^5+7x^3}$

52. Factoring. To factor a polynomial is to separate it into factors that cannot be further factored.

For example, to factor $ax + ay$ we see that a is a factor of each term, and that $ax + ay$ divided by a equals $x + y$. Therefore

$$ax + ay = a(x + y),$$

where the parentheses show that $x + y$ is to be multiplied by a. Hence the factors of $ax + ay$ are a and $x + y$.

Study also the following:

$$axy + axz = ax(y + z).$$
$$2a^2x - 4a^2y = 2a^2(x - 2y).$$
$$a^2b^3 + a^3b^2 = a^2b^2(b + a).$$
$$a^2 - b^2 = (a + b)(a - b), \text{ by § 49.}$$

Exercise 29. Factoring

Factor:

1. $ax - ay$.
2. $\pi a - \pi b$.
3. $2\pi a + 2\pi b$.
4. $\pi d - \pi d'$.
5. $\pi r^2 a - \pi r^2 b$.
6. $axy + axz$.
7. $ax^2y - axz$.
8. $\pi r^2 x - \pi r^2 y$.
9. $mnx^2 + mny^2$.
10. $2ax^2y - 4a^2xy^2$.
11. $3ax^2y + 9ax^2z$.
12. $7a^2b^2c^2 + 14abc$.
13. $25a^2b + 35ab^2$.
14. $125x^2y^2z^2 + 75xyz$.
15. $111abcd + 99a^2c^2$.
16. $p + prt$.
17. $p - prt$.
18. $\pi hr^2 + \pi kr^2$.
19. $\pi ma^2 + \pi nb^2$.
20. $\frac{1}{2}bh - \frac{1}{2}bk$.
21. $abc + 3a^2b^2c^2$.
22. $3a^2b + 3ab^2$.
23. $3a^2b - 3ab^2$.
24. $a^3 + 3a^2b + 3ab^2$.
25. $a^3 - 3a^2b + 3ab^2$.
26. $x^2 - y^2$.
27. $x^2 - 4$.
28. $9a^2 - b^2$.
29. $9a^2 - 4b^2$.
30. $144m^2 - 25n^2$.

53. Use of Factoring in simplifying Formulas. One of the uses of factoring is in the simplifying of formulas.

For example, suppose we wish to find the value of $\pi a - \pi b$ when $a = 325$ and $b = 324$. It is much easier to take $\pi(a - b) = \pi(325 - 324) = \pi = 3\frac{1}{7}$ than to multiply 325 by $3\frac{1}{7}$, and 324 by $3\frac{1}{7}$, and then subtract.

Exercise 30. Uses of Factoring

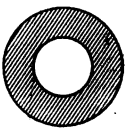
1. The interest on p dollars for 1 yr., the rate being r, is rp, and for t years it is trp; that is, $i = trp$. The amount of a note equals the principal plus the interest, or $a = p + trp$. Factor the right side of this equation. Find the value of a if $p = \$1250$, $t = 2\frac{1}{2}$, $r = 6\%$.

2. In finding the net price (n) of some goods we deduct from the marked price (m) a certain per cent (r) of the marked price; that is, $n = m - rm$. Factor this value of n. Find the value of n if $m = 750$ and $r = 30\%$.

3. In the right triangle of Ex. 11, p. 7, $z^2 = x^2 + y^2$. From this find x^2 in terms of z^2 and y^2. Factor this value.

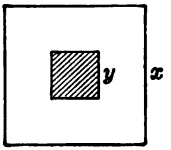
4. In Ex. 3, using the factored form, find the value of x if $z = 5$ and $y = 3$; if $z = 25$ and $y = 15$.

5. If the radius of the outer circle in this figure is x, and that of the inner circle is y, the area of the shaded ring is $\pi(x^2 - y^2)$. Factor this expression. From the factored result find the area when $\pi = 3\frac{1}{7}$, $x = 14$, and $y = 7$.



6. Show that the usual formula for the area of a trapezoid is derived by factoring $a = \frac{1}{2}bh + \frac{1}{2}bh'$.

7. If from this square that is x inches on a side a square is cut that is y inches on a side, how many square inches will be left? Factor the result and find the value when $x = 16$ and $y = 8$; when $x = 20$ and $y = 19$.



CHAPTER V

FRACTIONS

54. Fractions in Algebra. Algebraic fractions are treated in the same manner as fractions in arithmetic.

For example, $\frac{2}{15}+\frac{11}{15}=\frac{13}{15}$, and $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$;

$$\frac{21}{25}-\frac{19}{25}=\frac{2}{25}, \text{ and } \frac{x}{m}-\frac{y}{m}=\frac{x-y}{m};$$

$$\frac{2}{3}\times\frac{4}{5}=\frac{8}{15}, \text{ and } \frac{a}{b}\cdot\frac{x}{y}=\frac{ax}{by};$$

$$\frac{2}{3}\div\frac{5}{7}=\frac{2}{3}\times\frac{7}{5}=\frac{14}{15}, \text{ and } \frac{a}{b}\div\frac{x}{y}=\frac{a}{b}\cdot\frac{y}{x}=\frac{ay}{bx}.$$

55. Mixed Quantities reduced to Fractions. Required to express $a-b-\frac{a-b^2}{a+b}$ as a fraction.

Since $$1=\frac{a+b}{a+b},$$

therefore $$a-b=\frac{(a-b)(a+b)}{a+b}=\frac{a^2-b^2}{a+b}.$$

Therefore

$$a-b-\frac{a-b^2}{a+b}=\frac{a^2-b^2}{a+b}-\frac{a-b^2}{a+b}$$

$$=\frac{a^2-b^2-(a-b^2)}{a+b}$$

$$=\frac{a^2-b^2-a+b^2}{a+b}=\frac{a^2-a}{a+b}.$$

We therefore see that we proceed in algebra exactly as we proceed in a similar case in arithmetic.

Exercise 31. Mixed Quantities

Express in fractional form:

1. $2\frac{1}{2}$.
2. $3\frac{2}{3}$.
3. $a + \frac{1}{b}$.
4. $m + 1 + \frac{1}{m}$.
5. $m - \frac{1}{m}$.
6. $x + y + \frac{1}{2}$.
7. $a + \frac{m}{m+n}$.
8. $m - 1 - \frac{1}{m}$.
9. $a + \frac{b}{xy}$.
10. $x^2 + \frac{x^3}{x+y}$.
11. $x - y - \frac{xy}{y}$.
12. $a + b - \frac{a^2+b^2}{a-b}$.

56. Reducing Fractions to Lowest Terms. To reduce a fraction to lowest terms we proceed as in arithmetic.

Thus
$$\frac{18}{30} = \frac{3 \cdot \cancel{2} \cdot \cancel{3}}{5 \cdot \cancel{2} \cdot \cancel{3}} = \frac{3}{5};$$
$$\frac{a^2b^3}{ab^7} = \frac{a\cancel{b^3}}{\cancel{b^3}b^4} = \frac{a}{b^4};$$
$$\frac{a+b}{a^2-b^2} = \frac{\cancel{(a+b)}}{\cancel{(a+b)}(a-b)} = \frac{1}{a-b}.$$

Exercise 32. Reducing Fractions

Reduce to lowest terms:

1. $\frac{x^2y^3}{x^3y^2}$.
2. $\frac{2x^4y}{4x^3y^5}$.
3. $\frac{3x^3y}{5xy^3}$.
4. $\frac{12a^2x^3}{16ax^7}$.
5. $\frac{15a^5b^6}{25a^6b^6}$.
6. $\frac{-a^2b^3}{a^4b^5}$.
7. $\frac{a^7b^8}{-a^5b^{10}}$.
8. $\frac{-a^6b^8}{-a^{10}b^{10}}$.
9. $\frac{-4x^5y}{-6x^6y}$.
10. $\frac{-30x^5}{-40x^7}$.
11. $\frac{(a+b)c}{(a+b)d}$.
12. $\frac{ac+bc}{ad+bd}$.
13. $\frac{ac-bc}{ad-bd}$.
14. $\frac{a^2-b^2}{a-b}$.
15. $\frac{x^2-4}{x+2}$.

57. Lowest Common Denominator. The denominator of lowest degree to which several fractions may be reduced is called their *lowest common denominator* (*L. C. D.*).

For convenience it is customary to speak of reducing fractions to a common denominator, this being a short way of saying that they are reduced to fractions having a common denominator.

Reduce $\frac{a}{bc}$ and $\frac{x}{cp + cq}$ to fractions having the lowest common denominator.

Since the factors of the denominators are b, c, $p + q$, the L. C. D. is $bc(p + q)$.

Multiplying the terms of $\frac{a}{bc}$ by $p + q$, we have $\frac{a(p + q)}{bc(p + q)}$.

Multiplying the terms of $\frac{x}{cp + cq}$ by b, we have $\frac{bx}{bc(p + q)}$.

These fractions are equivalent to the given fractions and have the L. C. D. $bc(p + q)$.

Exercise 33. Lowest Common Denominator

Reduce to fractions having the lowest common denominator:

1. $\frac{1}{a}, \frac{a}{a + b}$.
2. $\frac{1}{m}, \frac{m}{m + 1}$.
3. $\frac{a}{bc}, \frac{b}{ac + c^2}$.
4. $\frac{p^2 + 4}{p^2 - 4p}, \frac{q}{p}$.
5. $\frac{a}{xy}, \frac{b}{y^2 + y}$.
6. $\frac{a^2 + 1}{a^3 - a}, \frac{1}{a^2}$.
7. $\frac{m}{n^2}, \frac{1}{mn + n^2}$.
8. $\frac{x}{p^3q^2}, \frac{y}{pq - p^2q^2}$.
9. $\frac{2pr}{2pr - r^2}, \frac{p}{r^2}$.
10. $\frac{m^2 + 1}{m^2 + m}, \frac{1}{m^3}$.
11. $\frac{x}{yz}, \frac{z}{y(x + y)}$.
12. $\frac{p}{q^2r^2}, \frac{1}{qr + pqr}$.
13. $\frac{m - 1}{m}, \frac{m}{m - 1}$.
14. $\frac{x^2 - 2xy + y^2}{x^2 + xy}, \frac{x^2 + 2xy + y^2}{xy}$.
15. $\frac{x^3 + 3x^2y + 3xy^2 + y^3}{x}, \frac{x^3 - 3x^2y + 3xy^2 - y^3}{y}$.

58. Addition of Fractions. Required to add the fractions $\frac{a}{b}$ and $\frac{b}{b+c}$.

The lowest common denominator is evidently $b(b+c)$.

$$\frac{a}{b} = \frac{(b+c)a}{(b+c)b} = \frac{ab+ac}{b^2+bc},$$

and

$$\frac{b}{b+c} = \frac{b^2}{(b+c)b} = \frac{b^2}{b^2+bc}.$$

Therefore the sum $= \frac{ab+ac+b^2}{b^2+bc}$.

Exercise 34. Addition

1. Add $\frac{1}{2}$ and $\frac{1}{4}$; $\frac{3}{8}$ and $\frac{1}{2}$; $\frac{2}{a}$ and $\frac{4}{a}$.

2. Add $\frac{a}{2}$ and $\frac{a}{3}$; $\frac{a}{b}$ and $\frac{c}{b}$; $\frac{a}{b}$ and 1.

3. Add $\frac{a}{a+b}$ and $\frac{b}{a+b}$; $\frac{a+b}{c+d}$ and $\frac{a-b}{c+d}$.

4. $\frac{1}{x} + \frac{1}{x+y}$.

5. $\frac{m}{m^2+mn} + \frac{1}{m}$.

6. $\frac{1}{a} + \frac{1}{b} + \frac{a-b}{ab}$.

7. $\frac{a}{b(a+b)} + \frac{b}{a+b}$.

8. $\frac{a+b}{a-b} + \frac{b}{a}$.

9. $\frac{x}{x^2-1} + \frac{1}{x}$.

10. $\frac{m}{n} + \frac{1}{m+n}$.

11. $\frac{p^2+pq}{p(p+q)^2} + \frac{p-q}{p}$.

Solve the equations:

12. $\frac{x}{2} + \frac{x}{3} = \frac{x}{6} + 120$.

13. $18 - \frac{x}{3} = 27 - \frac{x}{0.3}$.

14. $\frac{x}{3} + \frac{x}{5} = \frac{x}{30} + 141$.

15. $\frac{x+1}{5} + \frac{x-4}{5} = \frac{x}{3}$.

59. Subtraction of Fractions. Required to subtract $\frac{a-b}{a+b}$ from $\frac{a}{b}$.

The lowest common denominator is evidently $b(a+b)$.

$$\frac{a}{b} = \frac{(a+b)a}{(a+b)b} = \frac{a^2+ab}{ab+b^2},$$

and
$$\frac{a-b}{a+b} = \frac{b(a-b)}{b(a+b)} = \frac{ab-b^2}{ab+b^2}.$$

The difference
$$= \frac{a^2+ab-(ab-b^2)}{ab+b^2}$$
$$= \frac{a^2+ab-ab+b^2}{ab+b^2} = \frac{a^2+b^2}{ab+b^2}.$$

Exercise 35. Subtraction

1. $\frac{1}{x} - \frac{1}{x+y}$.
2. $\frac{p}{q} - \frac{p-q}{p-1}$.
3. $\frac{1}{p+q} - \frac{3}{pq}$.
4. $\frac{a}{bc} + \frac{b}{ca} - \frac{1}{abc}$.
5. $\frac{p+q+r}{p-q} - \frac{p+q}{p}$.
6. $\frac{1}{x-1} - \frac{1}{x}$.
7. $\frac{ax}{by} - \frac{a-x}{b-y}$.
8. $\frac{a+b^2}{b} - \frac{a}{a^2-b}$.
9. $\frac{a-b}{(a+b)^2} - \frac{a+b}{(a+b)^2}$.
10. $\frac{a^2+a-1}{a+1} - \frac{a-1}{a}$.

Solve the equations:

11. $\frac{x}{2} - \frac{x}{3} = 5$.
12. $\frac{x}{2} - \frac{x}{7} = x - 9$.
13. $\frac{x}{3} - \frac{x}{5} = 4$.
14. $\frac{x}{3} - \frac{x}{7} = x - 17$.

15. What number diminished by 3 equals the sum of the half and fifth of the number?

60. Multiplying a Fraction. We multiply fractions in algebra in the same way as in arithmetic. For example:

$$2 \times \frac{2}{5} = \frac{4}{5}. \qquad \frac{2}{3} \times \frac{4}{5} = \frac{8}{15}.$$

$$2 \times \frac{a}{b} = \frac{2a}{b}. \qquad \frac{2}{3} \times \frac{a}{b} = \frac{2a}{3b}.$$

$$2 \times \frac{3}{8} = \frac{3}{4}. \qquad \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}.$$

$$2 \times \frac{a}{2b} = \frac{a}{b}. \qquad \frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}.$$

$$a \times \frac{x}{ab} = \frac{x}{b}. \qquad \frac{a}{b} \times \frac{2b^2}{3a} = \frac{2b}{3}.$$

Exercise 36. Multiplication

Multiply:

1. $\frac{3}{5}$ by 7.
2. $\frac{3}{5}$ by a.
3. $\frac{3}{5}$ by 5.
4. $\frac{3}{5}$ by $5a$.
5. $\frac{3}{5}$ by $15a$.
6. $\frac{a}{b}$ by a.
7. $\frac{a}{b}$ by b.
8. $\frac{a}{b}$ by ab.
9. $\frac{a}{b}$ by $\frac{b}{a}$.
10. $\frac{a}{2b}$ by 4.
11. $\frac{a}{2b}$ by b.
12. $\frac{a}{2b}$ by $2b$.
13. $\frac{a}{2b}$ by $6b^2$.
14. $\frac{3a}{2b}$ by $\frac{12b^2}{a}$.
15. $\frac{a^2}{b^2}$ by $3a$.
16. $\frac{a^2}{b^2}$ by $3ab$.
17. $\frac{a^2}{b^2}$ by $\frac{b^2}{a^2}$.
18. $\frac{a^2}{b^2}$ by $\frac{b}{a}$.
19. $\frac{a+b}{b}$ by b.
20. $\frac{a+b}{2}$ by 2.
21. $\frac{a+b}{2b}$ by $4b$.
22. $\frac{a+b}{2b}$ by c.
23. $\frac{a+b}{2bc}$ by c.
24. $\frac{a-b}{2bc}$ by $2bc$.
25. $\frac{2a+b}{3}$ by $2a$.
26. $\frac{3a-b}{c}$ by $4a$.
27. $\frac{s^2+h^2}{2h}$ by $2h$.

61. Division of Fractions. Just as we divide 3 ft. by 2 in. by reducing to the same denominator (3 ft. ÷ 2 in. = 36 in. ÷ 2 in. = 18), so we may divide $\frac{3}{4}$ by $\frac{2}{3}$ by reducing to the same denomination ($\frac{3}{4} \div \frac{2}{3} = \frac{9}{12} \div \frac{8}{12} = 9 \div 8 = \frac{9}{8}$). But it is easier to see that $\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \cdot \frac{3}{2} = \frac{9}{8}$, and that

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc},$$

as explained in arithmetic.

Thus, to divide $\dfrac{4a^3 - 8a^2b}{6x^3 - 9x^2y}$ by $\dfrac{16a^4b^2}{15x^3y}$, we have, by factoring and multiplying by the reciprocal of the divisor,

$$\frac{15x^3y \cdot 4a^2(a - 2b)}{16a^4b^2 \cdot 3x^2(2x - 3y)} = \frac{5xy(a - 2b)}{4a^2b^2(2x - 3y)}.$$

Exercise 37. Division

1. Divide $\frac{1}{3}$ by 2; $\frac{1}{a}$ by b; $\frac{1}{x+y}$ by $x - y$.

2. Divide 1 by $\frac{1}{3}$; 2 by $\frac{1}{3}$; a by $\frac{1}{b}$; $m + n$ by $\frac{1}{m-n}$.

3. Divide 5 by $\frac{1}{3}$; 5 by 2; 5 by $\frac{2}{3}$; x by $\frac{a}{b}$; x by $\frac{a}{b+c}$.

4. Divide 5 by $\frac{2}{3}$; $\frac{5}{7}$ by $\frac{2}{3}$; $\frac{a}{b}$ by $\frac{c}{d}$. How do you divide one fraction by another?

5. $\dfrac{a}{b} \div \dfrac{a+b}{a-b}$.

6. $\dfrac{1}{x-1} \div \dfrac{y}{x}$.

7. $\dfrac{x^2+1}{x^2-x} \div \dfrac{1}{x}$.

8. $\dfrac{a}{3b} \div \dfrac{a+b}{b}$.

9. $\dfrac{2p}{21q} \div \dfrac{4p^2}{7q^2}$.

10. $\dfrac{a^7b}{a^7-b} \div \dfrac{ab^7}{b-a^7}$.

11. Divide the product of $\frac{a}{b}$ and $\frac{c}{d}$ by the product of $\frac{a}{d}$ and $\frac{c}{b}$. Divide the result by $\frac{1}{a}$.

62. Equations involving Multiplication. If we have the equation $13 = 4 + \frac{3}{x}$, we proceed as follows:

$$9 = \frac{3}{x}, \text{ by subtracting 4.}$$

$$9x = 3, \text{ by multiplying by } x.$$

$$x = \tfrac{1}{3}, \text{ by dividing by 9.}$$

This solution involves multiplication by x, which was not included in the treatment of equations on page 14.

Similarly, if we have the equation $\frac{x-1}{x+1} + \frac{2}{5} = 1$, we proceed as follows:

$$\frac{x-1}{x+1} = \frac{3}{5}, \text{ by subtracting } \frac{2}{5}.$$

$$5x - 5 = 3x + 3, \text{ by multiplying by } x + 1 \text{ and by } 5.$$

$$2x = 8, \text{ by subtracting } 3x \text{ and adding } 5.$$

$$x = 4, \text{ by dividing by } 2.$$

Exercise 38. Equations

Solve:

1. $\frac{3}{x} = 15.$
2. $\frac{15}{x} = 3.$
3. $\frac{x}{3} = 15.$
4. $\frac{x}{15} = 3.$
5. $\frac{75}{x} = 25.$
6. $\frac{111}{2x} = 3.$
7. $\frac{4}{x-1} = 1.$
8. $\frac{1}{x-1} = 4.$
9. $\frac{1}{2x-1} = \frac{1}{2}.$
10. $\frac{3}{2x+1} = 1.$
11. $\frac{x-1}{x+1} = \frac{3}{4}.$
12. $\frac{x+5}{x-5} = 3.$

13. If 20 is divided by a certain number increased by 1, the quotient is 10. What is the number?

14. If 7 more than a certain number is divided by 13 less than the number, the quotient is 3. What is the number?

63. Clearing of Fractions. Multiplying both members of an equation by such a number as to leave no fractions in the result (§ 62) is called *clearing an equation of fractions.*

To clear an equation of fractions, multiply both members by the least common multiple of the denominators.

Solve the equation

$$3 + \tfrac{2}{3}x = \tfrac{4}{5}x + 2.$$

Clearing of fractions by multiplying by 15,

$$45 + 10x = 12x + 30.$$

Subtracting both 45 and $12x$, so as to place all the x's on one side and all the known terms on the other,

$$10x - 12x = 30 - 45,$$

or

$$-2x = -15.$$

Dividing by -2 to find the value of x,

$$x = \tfrac{15}{2}, \text{ or } 7\tfrac{1}{2}.$$

Check. $3 + \frac{2}{3} \cdot \frac{15}{2} = \frac{4}{5} \cdot \frac{15}{2} + 2$; for each equals 8.

Exercise 39. Clearing of Fractions

Solve the equations:

1. $\dfrac{x}{3} + \dfrac{x}{2} = 7.$
2. $\dfrac{x}{5} + \dfrac{x}{3} = 20.$
3. $x + \dfrac{x}{3} = \dfrac{1}{3} + 7.$
4. $4x + \frac{1}{7} = \frac{1}{7}x + 4.$
5. $10 + 0.1x = 5 + \frac{1}{5}x.$
6. $\dfrac{2x}{5} + 3 = \dfrac{x}{3} + 4.$
7. $\dfrac{2x}{9} + \dfrac{x}{6} = \dfrac{x}{18} + \dfrac{1}{3}.$
8. $\dfrac{3x}{7} - \dfrac{1}{3} = \dfrac{x}{21} + 5.$
9. $3x + 4x = \frac{3}{4}x + \frac{1}{8}.$
10. $0.1x + 6.2 = 0.3x + 0.2.$

11. Find a number whose half, third, and fourth added together equals 36.

12. A man spends every year \$200 more than half his salary. In 3 years he saves \$900. How much is his salary?

64. The Formula as an Equation. A formula may be thought of as an equation, and one of the chief uses of algebra is the solution of equations of this nature.

For example, if the rate of interest is r, the interest for 1 yr. on p dollars is rp. Since this is the interest for 1 yr., for t years it is trp. Therefore

$$i = trp.$$

Here i stands for interest, t for time, r for rate, and p for principal.

If from this formula we wish to find t, we divide both sides of the equation by rp. Then

$$\frac{i}{rp} = t.$$

Likewise we may find r or p, thus: $r = \frac{i}{tp}$, and $p = \frac{i}{tr}$.

Exercise 40. Formula as an Equation

Given $i = trp$, find:

1. i, when $t = 2$, $r = 6\%$, $p = \$500$.
2. t, when $i = \$20$, $r = 4\%$, $p = \$250$.
3. r, when $i = \$52.50$, $t = 3$, $p = \$350$.
4. p, when $i = \$135$, $t = 5$, $r = 6\%$.
5. What principal, put at interest at 6%, will produce \$135 in 3 years?
6. How long will it take \$1250 to produce \$250 interest at 5%?
7. How long will it take \$2750 to produce \$1155 interest at 6%?
8. The formula for the amount (sum of principal and interest) is $a = p(1 + tr)$. From this formula deduce the formula for p.

9. Given the formula for the circumference of a circle, $c = 2\pi r$, find the formula for r.

10. If the circumference of a circle is 47.124 in., what is the radius? (Take $\pi = 3.1416$.)

11. Given the formula for the area of a circle, $a = \pi r^2$, find the formula for r^2; also the formula for r.

12. If the area of a circle is 154 sq. in., what is the radius? (Take $\pi = 3\frac{1}{7}$.)

13. Given the formula for the surface of a sphere, $s = 4\pi r^2$, find the formula for r^2; also the formula for r.

14. Given the formula for the volume of a sphere, $v = \frac{4}{3}\pi r^3$, find the formula for r^3; also the formula for r.

15. Given the formula for the volume of a cylinder, $v = h\pi r^2$, find the formula for h.

16. If the volume of a cylinder is 308 cu. in. and the radius is 7 in., what is the height? (Take $\pi = 3\frac{1}{7}$.)

17. If the volume of a cylinder is 1540 cu. in. and the height is 10 in., what is the radius? (Take $\pi = 3\frac{1}{7}$.)

18. Given the formula for the volume of a cone, $v = \frac{1}{3}h\pi r^2$, find the formula for h.

19. From the same formula, $v = \frac{1}{3}h\pi r^2$, find the formula for r^2; also the formula for r.

20. Given the formula for the area of a great circle of a sphere, $a = \pi r^2$, and the formula for the surface of a sphere, $s = 4\pi r^2$, find the ratio of a to s.

21. Given the formula for the volume of a cylinder, $v = h\pi r^2$, find the volume when $h = 2r$.

22. From Ex. 21 find the volume of a cylinder that is 2 in. high and 2 in. in diameter.

23. From the formula $v = h\pi r^2$ find the volume of a cylindrical shaft 6 ft. long and 4 in. in diameter.

24. If a train travels 80 mi. in 2 hr., what is its rate per hour? If it travels the distance d mi. in the time t hr., what is its rate per hour?

25. From the formula $r = \frac{d}{t}$ find r when $d = 80$, $t = 2$; when $d = 98$, $t = 2$; when $d = 134$, $t = 3$.

26. From the formula of Ex. 25 find the formula for d. Find the value of d when $r = 39$, $t = 4$.

27. From the formula of Ex. 25 find the formula for t. (First multiply by t and then divide by r.)

28. From the formula of Ex. 25 find the value of t when $r = 42$, $d = 105$.

29. Given the formula $V = \frac{4}{3}\pi r^3$, find the value of r^3 in terms of V, π, and the given fraction.

30. If $\frac{3V}{a} = \pi r^2$, find the value of V in terms of the other quantities. In the same formula find the value of r^2 in terms of the other quantities.

31. If $\frac{a}{b} = \frac{c}{d}$, find the value of a in terms of the other quantities; of b in terms of the other quantities.

32. Given the formula for the area of a triangle, $a = \frac{1}{2}bh$, find the formula for b; also the formula for h.

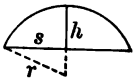
33. Given the formula for the area of a trapezoid, $a = \frac{1}{2}(b + b')h$, find the formula for h.

34. Given the formula of Ex. 33, find the value of h when $a = 32$, $b = 10$, and $b' = 6$.

35. Given the formula of Ex. 33, let $a = 66$, $b = 12$, $h = 6$, and substitute these values in the formula. Then find the value of b'.

36. In the same way as in Ex. 35 find the value of b when $a = 35$, $b' = 8$, and $h = 5$.

37. Carpenters have a formula for finding the radius of a circle used in constructing an arch of height h, the span being $2s$, as follows: $r = \frac{s^2 + h^2}{2h}$. Find the value of r when $s = 4$ and $h = 3$.



38. Using the formula of Ex. 37, find the value of r when $s = 8$ and $h = 2$.

39. Using the formula of Ex. 37, find the value of s when $r = 8\frac{1}{8}$ and $h = 6$.

40. The area of an equilateral triangle, each of whose sides is s, is expressed by the formula $a = \frac{1}{4}s^2\sqrt{3}$. Find the value of a to two decimal places when $s = 2$.

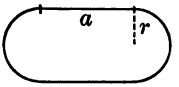
41. In the formula $a = \frac{1}{4}s^2\sqrt{3}$ multiply both of these equals by 4 and then divide both by $\sqrt{3}$.

42. The volume of a rectangular solid that is l long, w wide, and h high, is expressed by the formula $v = lwh$. Find the value of l in terms of v, w, and h.

43. A machinist finds in the *Scientific American* this formula: $p = r(2 + \pi)$. He wishes from this to obtain a formula for r. What is the formula?

44. A carpenter finds in a book this formula: $h^2 = a^2 + b^2$. He wishes from this to obtain a formula for a^2. What is the formula?

45. The formula for the distance around a running track with semicircular ends is $d = 2a + 2\pi r$. From this find a formula for r, so we may know the radius to be used in drawing the circles.



46. From the formula of Ex. 45 find the value of d when $a = 300$ and $r = 50$.

47. From the formula of Ex. 45 find the value of a when $d = 1114.16$ and $r = 50$. (Take $\pi = 3.1416$.)

65. Solving a Simple Equation. We have solved many simple equations in our work in algebra, and have found that we proceed as follows:

(1) *Transpose all terms involving x to one member, and all other terms to the other member.*

(2) *Unite the terms in each of the members.*

(3) *Divide both members by the coefficient of x.*

Thus, given

$$5x - 7 = 2x + 20.$$

$5x = 2x + 27$, by adding 7 (transposing -7).

$3x = 27$, by subtracting $2x$ (transposing $2x$).

$x = 9$, by dividing by 3.

Exercise 41. Equations Reviewed

Solve the following:

1. $3x + x = 64.$
2. $5x + 4x = 81.$
3. $9x - 3x = 78.$
4. $12x - 7x = 95.$
5. $17x - 9x = 96.$
6. $15x + 7 = 52.$
7. $13x + 9 = 100.$
8. $19x + 8 = 179.$
9. $23x - 7 = 108.$
10. $27x - 3 = 267.$
11. $4x + 7 = 3x + 64.$
12. $4x - 7 = 3x + 72.$
13. $7x + 3 = 3x + 67.$
14. $8x + 5 = 5x + 116.$
15. $9x - 4 = 3x + 68.$
16. $11x - 9 = 5x + 117.$
17. $24x - 7 = 9x + 158.$
18. $27 - 3x = 68 - 4x.$
19. $42 - 3x = 48 - 9x.$
20. $77 - 8x = 91 - 15x.$

21. From the formula $i = rtp$, find the value of p when $i = 70$, $t = 2\frac{1}{2}$, $r = 3\frac{1}{2}\%$.

22. The amount of a note is equal to the principal plus the interest. It is represented by the formula $a = p + rtp$. Find the value of p when $a = 1344$, $r = 4\%$, $t = 3$.

23. In the formula of Ex. 22 what does p equal when $a = \$224$, $r = 6\%$, and $t = 2$?

24. In the formula $a = p + rtp$ find the value of t. What does t equal when $a = 672$, $r = 4\%$, and $p = 600$?

25. How long will it take \$325 and interest to amount to \$373.75 at 5%?

26. In the formula $a = p + rtp$ find the value of r.

27. At what rate of interest will \$275 produce \$49.50 in 3 yr.?

28. A man lost 11% of his capital and then had \$6675. How much had he at first?

29. A man gained 11% on his capital and then had \$8325. How much had he at first?

30. A dealer sold some goods for \$2170, thus making a profit of 24%. What did the goods cost him?

31. A collection agency charges 4% for collecting a debt and remits \$1200. How much did it collect?

32. A bank charges 0.1% exchange on a draft. The entire cost of draft and exchange is \$1751.75. What is the face of the draft?

33. A piece of cloth lost 20% of its length in shrinking, and was then 72 yards long. How long was it originally?

34. In a machine shop the machine work on each casting of a certain kind was \$0.67. The machine work on a lot of castings cost \$871.75, which included an extra charge of \$17.50 for freight. How many castings were there?

35. The total weight of a 12-wheel passenger coach with the equipment for lighting it by electricity is 107,910 lb., of which $\frac{1}{33}$ is due to the electric-light equipment. What was the weight on each pair of wheels before the electric-light equipment was added?

36. On a certain road it is known that a passenger coach weighs 90% as much as a sleeping car, that the pay car weighs 88% as much as a passenger coach, and that the pay car weighs 39.6 tons. What does a sleeping car weigh?

37. A freight engine took on 7000 lb. of coal, and after $5\frac{1}{2}$ hr. it had 400 lb. left. How much did it use per hour?

38. A boy working in a shop from 7.30 A.M. to 5.15 P.M., with 45 minutes off for lunch, earned $1.35, which included an extra 10¢ for going on an errand at noon. What were his regular wages per working hour?

39. To make 14 steel frames for locomotive tenders a shop needed 57,800 lb. of steel beams (called "channels"), in which was included 120 lb. allowance for waste. Each tender used 4 channels, each 25 ft. 9 in. long. What was the weight of the channels per foot?

40. A passenger coach with passengers and baggage weighs 95,900 lb. The passengers weigh 5% of the weight of the coach, and the baggage weighs 1400 lb. Allowing 150 lb. as the average weight of the passengers, how many passengers are there?

41. An axle of a car was turned in a lathe and lost $6\frac{1}{4}$% of its weight. It then lacked $32\frac{1}{2}$ lb. of weighing just 1000 lb. What was its weight before it was turned?

42. How many revolutions per minute must be made by a wheel 10 in. in diameter so that the rim shall be traveling 523.6 ft. per minute? (Use 3.1416 for π.)

43. What is the length of a steel bar with a square cross section 2 in. wide, that weighs 245 lb.? (Steel weighs 490 lb. per cubic foot.)

44. A copper pipe 24 in. in diameter weighs 774.4 lb. It is made from sheet copper weighing 15.4 lb. per square foot. What is the length of the pipe? (Use $\frac{22}{7}$ for π.)

45. The sum of two numbers is 50, and the less is $\frac{2}{3}$ the greater. What are the numbers?

46. If 110 is divided by 1 less than a certain number, the result is $36\frac{2}{3}$. What is the number?

47. If 175 is divided by 3 less than five times a certain number, the result is $3\frac{34}{47}$. What is the number?

48. If to $\frac{1}{25}$ of the sum of 3 and three times a certain number there is added $\frac{1}{12}$ of the number, the result is 5. What is the number?

49. A man spent $150 more than half of his income each year for two years. He saved $800 in the two years. What was his annual income?

50. The distance around a certain rectangular field is 256 rd., and the length is 3 times the width. What is the width? the length? the area?

51. The profit to be divided among three partners is $10,000. B receives $2\frac{1}{3}$ times as much as A, and A receives 30% as much as C. How much does each receive?

52. The perimeter of a triangle is 8.1 in.; the shortest side is $\frac{1}{2}$ as long as the longest one, and the longest one is $33\frac{1}{3}$% longer than the third side. What is the length of each side?

53. Albany is $\frac{1}{3}$ of the way from New York to Buffalo, Rochester is $\frac{4}{5}$ of the way from Albany to Buffalo, and it is 60 miles from Rochester to Buffalo. Required the distance from New York to Buffalo.

54. A man started in business with a certain capital. He gained $1000 the first year, lost half of what he then had the second year, and gained $2000 the third year. He then found that he had the same amount with which he started. How much was it?

55. In preparing some medicine a druggist wishes to know how much water he must add to a quart of alcohol, which already contains 5% water, so that the mixture may contain 50% alcohol.

Let x = the *number* of quarts of water to be added.

Then $50\%(1 + x)$ = number of quarts of alcohol in the mixture, which must be the 95% of a quart with which he started, since no alcohol has been added.

Then
$$50\%(1 + x) = 95\%,$$
$$1 + x = \tfrac{95}{50},$$
$$x = \tfrac{95}{50} - 1 = \tfrac{45}{50} = \tfrac{9}{10}.$$

Therefore he must add $\frac{9}{10}$ of a quart of water.

56. How many ounces of gold must be melted with 30 oz. of gold 16 carats fine ($\frac{16}{24}$ pure) to make an ingot 18 carats fine?

57. How many ounces of pure gold must be melted with 18 oz. of pure gold and 6 oz. of silver to make an ingot 22 carats fine?

58. How much water must be added to a quart of a solution containing 8% acid and the rest water, so that the new mixture shall contain 6% acid?

59. How many ounces of pure silver must be melted with 300 oz. of silver 800 fine (800 parts of pure silver in 1000 parts of metal) to make a bar 850 fine?

60. How many ounces of pure silver must be melted with 400 oz. of silver and 100 oz. of tin to make a bar 900 fine?

61. How much water must be added to a gallon of a solution containing 9% of a certain extract, so that the new mixture shall contain 4% of extract?

62. How much cottonseed oil must be added to a pint of a mixture, which is $\frac{1}{4}$ cottonseed oil and the rest olive oil, so that the new mixture shall contain $\frac{7}{8}$ cottonseed oil?

66. Common Problems of Algebra. Besides the problems taken from daily life in the shops and in mercantile houses there are many interesting problems that are easily solved by algebra and that usually make up much of the algebraic work of the school. The following is an example:

Find a number whose seventh part minus its eleventh part equals 4.

Let $x =$ the number.

Then $\frac{x}{7} - \frac{x}{11} = 4.$

Clearing of fractions, as in § 63, by multiplying both members of the equation by 7×11, we have

$$11x - 7x = 308,$$

whence $4x = 308,$

and $x = 77.$

Exercise 42. Problems

1. Find a number whose third part plus 9 equals 25; whose third part minus 9 equals 25.

2. Find a number whose fifth part minus 1 equals 29; whose fifth part plus 1 equals 29.

3. Find a number such that if 7 is added to $\frac{2}{3}$ of it, the sum is 35.

4. Find a number whose fourth part minus its fifth part equals 2.

5. Find a number such that $\frac{5}{7}$ of it less $\frac{3}{8}$ of it equals 19.

6. What number increased by $\frac{4}{5}$ of itself equals 99?

7. Half the remainder found by subtracting 14 from a certain number equals a fourth of the sum of the number and 14. What is the number?

8. Find a number whose half, third, and fourth parts added together equals 39.

9. After selling $\frac{1}{2}$ of his farm, and then $\frac{1}{8}$ of what was left, a man still had 140 acres. How many acres had he at first?

10. There is a certain number such that its fourth part added to its fifth part equals one less than its half. What is the number?

11. The sum of two numbers is 90, and one is 80% of the other. What are the numbers?

12. The sum of two numbers is 100, and the less is $\frac{2}{3}$ the greater. What are the numbers?

13. If 220 is divided by one less than a certain number, the result is 55. What is the number?

14. Find three consecutive numbers whose sum is 87. (Represent the numbers by $x - 1$, x, and $x + 1$.)

15. A tree 60 feet high is broken so that the part broken off is four times the length of the part left standing. What is the length of each part?

16. A tree 84 ft. high is broken so that the part broken off is five times the length of the part left standing. What is the length of each part?

17. Distribute $36 among four men so that the first two shall each receive twice as much as each of the second two.

18. If we add 24 to a certain number, the sum is as much above 80 as the number is below 80. What is the number?

19. A farmer bought 18 sheep. If he had bought 3 more for the same money, each sheep would have cost him $1 less. How much did he pay for each sheep?

20. Two men buy an automobile for $2500. One pays $100 more than twice what the other pays. If they use the automobile in proportion to what they pay, how many days should each use it out of every 25 days?

67. Easy Equations involving Two Unknown Quantities. In solving a problem it is sometimes convenient to employ two unknown quantities.

1. Find two numbers whose sum is 12 and whose difference is 6.

Let $x =$ the larger number,

and $y =$ the smaller number.

Then $x + y = 12$, since their sum is 12, (1)

and $x - y = 6$, since their difference is 6. (2)

Adding (1) and (2), $2x = 18$.

Dividing by 2, $x = 9$.

Substituting 9 for x in (1),

$$9 + y = 12.$$

Subtracting 9, $y = 3$.

Therefore the two numbers required are 9 and 3.

2. Find two numbers such that three times the first plus twice the second is 20, and five times the first minus the second is 3.

Let $x =$ the first number,

and $y =$ the second number.

Then, from the statement of the problem,

$$3x + 2y = 20, \quad (1)$$

and

$$5x - y = 3. \quad (2)$$

We see that if we multiply both members of (2) by 2, we can add (1) to this result and the y's will disappear. Therefore, multiplying (2) by 2,

$$10x - 2y = 6. \quad (3)$$

Adding (1) and (3), $13x = 26$.

Dividing by 13, $x = 2$.

Substituting 2 for x in (1),

$$6 + 2y = 20.$$

Subtracting 6, $2y = 14$.

Dividing by 2, $y = 7$.

Therefore the two numbers required are 2 and 7.

Exercise 43. Two Unknowns

1. The sum of two numbers is 88, and one number is 20 more than the other. What are the numbers?

2. The sum of the ages of a father and son is 84 yr., and the father is twice as old as the son. How old is the son?

3. The distance around a certain rectangular field is 78 rd., and the length is 2.9 times the width. What is the width? the length?

4. A dealer bought 25 dozen oranges for $9.25. For a part he paid 40 cents a dozen and for the remainder 35 cents a dozen. How many dozen of each did he buy?

5. A boy is three times as old as his sister. A year ago he was four times as old as she. How old are they now? Three years hence he will be how many times as old as she?

6. The age of one boy is $2\frac{1}{2}$ times that of another, and the sum of their ages is 21 yr. Find the age of each.

7. One number is four times as large as another. If I take the smaller number from 12 and the larger from 21, the remainders are equal. What are the numbers?

8. The sum of the ages of a father and son is 50 yr. If the age of the son is doubled, the result is 17 yr. less than the age of the father. What is the age of each?

9. In a class of 29 pupils there are 3 more girls than boys. How many are there of each?

10. At an election there were two candidates, and 3478 votes were cast. The successful candidate had a majority of 436. How many votes were cast for each?

11. One man has twice as much money as another; but if the second gives the first $10, the first will then have $3\frac{1}{2}$ times as much as the second. How much has each?

CHAPTER VI

PROPORTION

68. Ratio. The relation of one quantity to another of the same kind, as expressed by division, is called the *ratio* of the first to the second.

69. Proportion. An expression of the equality of two ratios is called a *proportion.*

The ratio of 2 in. to 5 in. may be indicated thus: $\frac{2 \text{ in.}}{5 \text{ in.}}$, or 2 in. : 5 in. The equality of this ratio to the ratio of 4 ct. to 10 ct. may be indicated thus: $\frac{2 \text{ in.}}{5 \text{ in.}} = \frac{4 \text{ ct.}}{10 \text{ ct.}}$, or 2 in. : 5 in. = 4 ct. : 10 ct.

70. A Proportion is an Equation. The proportion $\frac{x}{2} = \frac{14}{4}$, or $x : 2 = 14 : 4$, is merely a simple equation.

71. Extremes and Means. In a proportion the first and last terms are called *extremes,* and the second and third the *means.*

Exercise 44. Proportion

1. Write the proportion $x : a = b : c$ in fractional form.

2. Using this fractional form, clear of fractions and show that $xc = ab$.

3. In the same way, using the fractional form, show that $x = \frac{ab}{c}$.

4. From Ex. 2 write out a statement concerning the product of the extremes equaling some other product.

72. Relation of Extremes to Means. It was proved in Exs. 2 and 3 on page 71 that, in any proportion,

(1) *The product of the extremes equals the product of the means.*

(2) *The product of the means divided by one extreme equals the other extreme.*

73. Terms may be considered Abstract. In a proportion involving cents and inches it is of course impossible to multiply inches and cents together. But because

$$\frac{2 \text{ in.}}{3 \text{ in.}} = \frac{2}{3}, \text{ and } \frac{5 \text{ ct.}}{7\frac{1}{2} \text{ ct.}} = \frac{5}{7\frac{1}{2}}, \text{ we see that}$$

The terms of a proportion may all be considered as abstract.

Solve the proportion $x : 9 = 34 : 153$.

Writing this $\frac{x}{9} = \frac{34}{153}$, we have a simple equation.

Therefore $$x = \frac{\cancel{9} \cdot 34}{\underset{17}{\cancel{153}}} = 2.$$

Solve the proportion $12 : x = 27 : 18$.

That is, $$\frac{12}{x} = \frac{27}{18} = \frac{3}{2}.$$

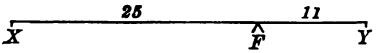
Clearing of fractions, $24 = 3x$, and $8 = x$.

Exercise 45. Proportion

Solve the following:

1. $\frac{x}{84} = \frac{9}{63}$.
2. $\frac{51}{21} = \frac{x}{7}$.
3. $19 : 33 = x : 231$.
4. $\frac{37}{x} = \frac{259}{77}$.
5. $\frac{245}{75} = \frac{98}{x}$.
6. $x : 125 = 25 : 625$.

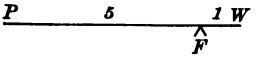
74. Lever and Fulcrum. If we take a yardstick and balance it, as shown in the figure, with weights at X and Y, we have a *lever*. The point F is called the *fulcrum*.



75. Law of the Lever. Now if we put F 25 in. from X, it will be 11 in. from Y, because the stick is 36 in. long. We then find that the weight x at X will have to the weight y at Y the ratio 11 : 25; that is, $\frac{x}{y} = \frac{11}{25}$. More generally, if a is the distance from F to X, and b is the distance from F to Y, then $\frac{x}{y} = \frac{b}{a}$, or $ax = by$.

This law, easily proved in class, is called the *law of the lever*.

For example, if a 6-foot lever has a fulcrum 1 ft. from the weight W, and if P is the power we must use to lift W, we know that $\frac{W}{P} = \frac{5}{1}$; that is, $P = \frac{1}{5} W$. Hence to lift 100 lb. we need exert a power of only 20 lb.



Exercise 46. The Lever

1. Where must we place the fulcrum under a 12-foot plank that a 56-pound boy may balance one who weighs 112 lb.?

2. A father puts the fulcrum 2 ft. from his end of a 10-foot plank and just balances his son, who weighs 40 lb. How much does the father weigh?

3. Two boys balance a seesaw, the plank being 12 ft. long. The fulcrum is 5 ft. from the heavier boy, who weighs 105 lb. How much does the other weigh?

4. If Rob has an iron bar $4\frac{3}{4}$ ft. long, and wishes to pry up a 300-pound rock, and he weighs 80 lb., how far from the stone must he place the fulcrum, making no allowance in this style of problem for taking hold of the bar or reaching under the stone?

5. A man weighing 180 lb. wishes to raise a piece of rock weighing 720 lb. by the help of a crowbar 5 ft. long. What is the greatest distance at which the fulcrum can be placed from the end of the bar that is applied to the stone?

6. If the fulcrum of a 5-foot crowbar is 6 in. from the end, what weight can be lifted by a man weighing 200 lb.?

7. If the fulcrum of a 6-foot crowbar is 7 in. from the end, what weight must be applied to lift 800 lb.?

8. If a pressure of 10 tons is exerted at a point 6 in. from the center of a drive wheel of a machine, what pressure is exerted at the rim, the radius being 2 ft.?

9. A power of 50 lb. is exerted on one of the spokes of a wheel 3 in. from the center. The spoke being 20 in. long, what force does this represent at the circumference?

10. A windlass has a radius of 3 in., and is operated by a wheel with a radius of 2 ft. If we wish to raise a weight of 100 lb., what power must we exert at the end of the radius of the wheel?

11. A door is 3 ft. wide. A boy tries to close it by pushing upon it at the outer edge. Another boy pushes on the other side of the door, halfway from the hinge to the outer edge, exerting a pressure of 50 lb. How much pressure must the first boy exert before the door will begin to close?

12. A pair of pincers is grasped 6 in. from the hinge, and a piece of wire is inserted $\frac{3}{4}$ in. from the hinge. A squeezing force of 6 lb. is applied by the hand. What force is exerted on the wire? (Notice that the hinge is a fulcrum, and that either arm of the pincers is a lever.)

13. A nutcracker is so held that the hand exerts a pressure of 4 lb. at points exactly 5 in. from the hinge. The pressure on the nut is exerted $1\frac{1}{8}$ in. from the hinge. How much pressure is exerted on the nut?

76. Inverse Ratio. If it takes 20 days for 15 men to do a piece of work, how long will it take 10 men to do it?

The ratio of the times is $\frac{x}{20}$.

Because 10 men do it in x days and 15 men in 20 days, therefore the ratio of the men is $\frac{10}{15}$, and the *inverse ratio* is $\frac{15}{10}$.

Since twice as many men will do the work in half the time, $\frac{10}{15}$ as many men will do it in $\frac{15}{10}$ of the time.

Therefore $\frac{x}{20} = \frac{15}{10}$,

and $x = \frac{20 \cdot 15}{10} = 30$. Therefore it takes 10 men 30 days.

Exercise 47. Inverse Ratio

1. If it takes you 4 min. to clean a blackboard, how long will it take two of the class at the same rate?

2. I have enough grain to last my 2 horses 3 months. If I buy another horse, how long will the grain last?

3. With a certain quantity of wool a man can make 52 yd. of goods 24 in. wide. How many yards can he make if it is 1 yd. wide?

4. It takes 56 yd. of 27-inch carpet for my parlor. How much will be needed if I use carpet 1 yd. wide, assuming that it will cut as economically?

5. There were two pieces of sodding of the same size. It took 3 men 5 days to do the first piece. How long will it take 2 men to do the second?

6. A builder had 4 plasterers at work 12 days in plastering one of the stories of an apartment house. How long will it take 6 men to plaster the next story? If the builder puts 8 men at work on the next story, how long will it take them? (The rooms are the same for all stories.)

77. Similar Figures. Figures which have exactly the same shape are called *similar figures.*

For example, two circles are similar figures; also two squares, two equilateral triangles, two cubes, or two spheres.

1. What is the area of a square 2 in. on a side? also of one that is twice as long on a side? (Draw the pictures if necessary.)

2. The diagonal of a square 1 in. on a side is 1.4 in. How long is the diagonal of a square 2 in. on a side? (If in doubt, measure it.)

3. What is the volume of a cube that is 1 in. on an edge? also of one that is 2 in. on an edge? (Use inch cubes if necessary.)

4. If one side of an equilateral triangle is 3 in., what is the perimeter of another equilateral triangle that is twice as high?

78. Proportion related to Similar Figures. We may infer from the above that, in similar figures,

(1) *Corresponding lines are proportional.*

That is, if the radius of one circle is twice that of another, the circumference of the one is twice that of the other.

(2) *Areas are proportional to the squares of corresponding lines.*

That is, if one equilateral triangle is twice as high as another, the area is 2^2, or 4, times that of the other.

(3) *Volumes are proportional to the cubes of corresponding lines.*

That is, if the radius of one sphere is twice that of another, the volume of the one is 2^3, or 8, times that of the other.

79. Exercises in Similar Figures. If the area of a circle is 3.8 sq. in., what is the area of a circle of twice the diameter?

From § 78 we have the proportion

$$x : 3.8 = 2^2 : 1,$$

or

$$\frac{x}{3.8} = \frac{4}{1}.$$

Multiplying by 3.8, $x = 15.2.$

Therefore the area is 15.2 sq. in.

Exercise 48. Similar Figures

1. If $\frac{1}{4}$ in. on a map represents 100 mi., how many miles will $1\frac{1}{4}$ in. represent?

2. If a tree casts a shadow 50 ft. long when a post 4 ft. high casts a shadow 5 ft. long, how high is the tree?

3. If a distance of 325 mi. is represented on a map by $1\frac{5}{8}$ in., how many inches will represent 340 mi.?

4. If a distance of 825 mi. is represented on a map by $2\frac{1}{4}$ in., how many inches will represent $206\frac{1}{4}$ mi.? $412\frac{1}{2}$ mi.? $103\frac{1}{8}$ mi.? $1237\frac{1}{2}$ mi.?

5. A photograph in which a house appears as 1.7 in. high, and a tree as 1.5 in. high, is enlarged so that the house appears as 2.1 in. high. How high does the tree appear?

6. A triangle has its sides 3 in., 4 in., 5 in. Another triangle of the same shape has its shortest side 2 in. What are the lengths of the other sides?

7. Of the two triangles in Ex. 6, the first has an area of 6 sq. in. What is the area of the second?

8. A cylindrical can holds a pint. How much will a similar one hold if it is $1\frac{1}{2}$ times as high?

9. A box 3 in. long has a volume of 10.5 cu. in. What is the volume of a box of the same shape, 4 in. long?

10. A toy balloon 6 in. in diameter contains $113\frac{1}{7}$ cu. in. If it is inflated to 7 in. in diameter, what will be the volume?

11. A certain projectile for the gun of a battleship weighs 650 lb. What is the weight of a similar one of which the length is 10% more?

12. Of the two projectiles mentioned in Ex. 11, the area of the surface of the second is how many times that of the first?

13. If a post 8 ft. high casts a shadow $\frac{2}{15}$ as long as that cast at the same time by a tree, how high is the tree?

14. If a building casts a shadow 60 ft. long when a man 6 ft. high casts a shadow 9 ft. long, how high is the building?

15. A man whose eye is 5 ft. 6 in. above the ground sights over the top of a 12-foot pole and just sees the top of a tree. If he is 7 ft. from the pole and 63 ft. from the tree, how high is the tree? Draw a plan illustrating this example.

16. The class has considered the question of a model kitchen in an apartment house. The members drew a plan to the scale $\frac{1}{60}$, representing the kitchen as 3 in. long and 2 in. wide. What was the floor space of the proposed kitchen?

17. A plan for a rectangular flower bed is drawn to the scale $\frac{1}{80}$. The plan is 3 in. by 4.5 in. What is the perimeter of the bed? How many square feet does it contain?

18. This figure represents a pair of proportional compasses used by draftsmen. By adjusting the screw at O, the lengths OA and OC, and the corresponding lengths OB and OD, may be varied proportionally. If $OA = 3$ in. and $OC = 5$ in., then AB is what part of CD?

19. A class wishing to find the height of the school building, one of the boys stood a yard measure upright and another lay on the ground where he could just see the top of the building in line with the top of the stick. The distance from his eye to the foot of the stick was 4 ft. 6 in., and to the building 45 ft. How high was the building? Draw a plan.

CHAPTER VII

MISCELLANEOUS APPLICATIONS

80. Nature of the Applications. The algebra commonly needed in the shop or in business has now been studied. The following miscellaneous applications apply the principles already considered.

Teachers will find it of value to have pupils bring into class any formulas of trade or the shop that may be used in their particular localities, provided they are within the range of knowledge of the class. The most common formulas and applications that appeal generally to a class have, however, been given in this work.

A pupil should not be required to solve a problem by algebra if he has a satisfactory solution by arithmetic. It should be remembered that there is no exact dividing line between the two subjects.

Exercise 49. Miscellaneous Applications

1. There is a certain law of machines expressed by the formula $\frac{F}{R} = \frac{d'}{d}$. From this find the value of F in terms of the other three letters; also the value of R in terms of the other three letters.

2. The law of velocity of a moving object is expressed by the formula $v = \frac{s}{t}$, where v stands for the uniform or average velocity, s for the space passed though, and t for the time. From this find the value of t in terms of v and s.

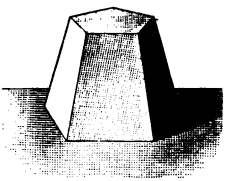
3. There is a certain law of the pendulum that is expressed by the formula $g = l \times \left(\frac{3.1416}{t}\right)^2$. From this find the value of l in terms of numbers and the other letters.

4. There is a certain law of moving bodies that is expressed by the formula $v - v' = at$. From this find the value of t in terms of the other letters; of v' in terms of the other letters.

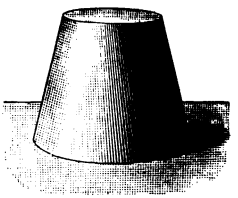
5. There is another law of moving bodies that is expressed by the formula $s = t \times \frac{v + v'}{2}$. From this find the value of t in terms of a number and the other letters.

6. When a projectile weighing P pounds is fired with a muzzle velocity of V feet per second from a gun weighing W pounds, the velocity at which the gun recoils is indicated by the formula $R = \frac{1}{W} \cdot VP$. Find the value of R when $W = 50{,}000$, $V = 2300$, and $P = 500$. What does the value of R mean?

7. This figure represents a solid called a *frustum of a pyramid.* It is shown in geometry that its volume is expressed by the formula $v = \frac{1}{3} h(b + b' + \sqrt{bb'})$, where h represents the height, and b and b' represent the areas of the bottom and the top, the two *bases* as they are called. Find the value of v when $h = 9$, $b' = 2$, and $b = 8$.



8. This figure represents a solid called a *frustum of a cone.* It is shown in geometry that its volume is expressed by the formula $v = \frac{1}{3} \pi h(r^2 + R^2 + rR)$, where h represents the height, r the radius of the smaller circle (the upper base), and R the radius of the larger circle (the lower base). Taking $3\frac{1}{7}$ as the value of π, find the value of v when $h = 14$, $r = 3$, and $R = 5$.



9. From the formula of Ex. 8 find the value of v when $h = 21$, $r = 5$, and $R = 2r$.

10. One man can do a piece of work in 8 days and another man can do it in 10 days. How many days will it take the two together to do the same work, at the same rate?

11. A water tank can be filled by three pipes, running separately, in 4 hours, 5 hours, and 6 hours respectively. In how many hours will the empty tank be filled by all three running together?

12. Bronze contains 9 parts of copper by weight to 2 parts of tin. How many pounds of tin in a bronze statue that weighs 2662 lb.?

13. A farmer finds that a bin 8 ft. long, 3 ft. 6 in. wide, and 5 ft. deep holds 112 bu. How many bushels in a bin that is 25% longer, 25% narrower, and has the same depth?

14. Water is flowing into a rectangular tank whose base is 6 ft. 8 in. long and 4 ft. 6 in. wide, at the rate of 2 cu. ft. in 3 min. How long will it take the water to fill the tank to a depth of 6 ft.?

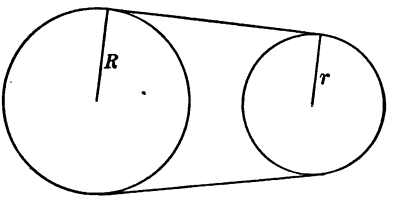
15. A manufacturer sells brass lamps to a dealer at such a price that the dealer can sell them at \$6.60 and make a profit of 32%. The manufacturer himself makes a profit of 25%. What is the cost of manufacture per lamp?

16. On which will the manufacturer make the greater per cent of profit: an article which costs \$2.90 to make, and which he sells for \$3.33$\frac{1}{3}$ less 3%, or one costing \$8.70 to make, which he sells for \$12 less $\frac{1}{8}$? How much greater is this per cent of profit?

17. Brass shrinks in cooling 1.5625% of the length of the mold into which it is poured. In casting brass hinges how long must be the mold for a hinge that is to be $5\frac{7}{8}$ in. long? for a hinge that is to be $11\frac{3}{4}$ in. long?

18. There is a law of falling bodies that is expressed by the formula $s = \frac{1}{2}at^2$. From this find the value of a in terms of a number and the other two letters.

19. If two wheels are belted together, one with radius R making N revolutions per minute, and the other with radius r making n revolutions per minute, then $2\pi RN = 2\pi rn$. From this find the value of n in terms of R, N, and r. If $R = 21''$, $r = 9''$, and $N = 107\frac{1}{7}$, what is the value of n?



20. When a locomotive goes around a curve it tends to run off the track, exerting a horizontal pressure on the rail indicated by the formula $p = \frac{WV^2}{32r}$, where W is the weight of the locomotive in pounds, V is its velocity in feet per second, and r is the radius of the curve in feet. If a locomotive weighing 60 tons runs around a curve 1500 ft. in radius at the rate of 50 mi. an hour, what is the horizontal pressure on the track?

21. If an engine can raise F pounds through S feet in T minutes, the horse power (H.P.) is indicated by the formula $\text{H.P.} = \frac{F \times S}{T \times 33{,}000}$. What is the H.P. of an engine that raises 20 tons through 50 ft. in 2 min.?

22. If a is the area of a piston in square inches, p the pressure of the steam in pounds, l the length of the piston stroke in feet, and n the number of strokes per minute, the horse power of an engine is indicated by the formula $\text{H.P.} = \frac{plan}{33{,}000}$. Find the H.P. when $p = 110$, $l = 2\frac{1}{2}$, $a = 288$, and $n = 100$. If the H.P. is 480, and if $l = 2$, $a = 250$, and $n = 120$, what is the steam pressure?

23. A field is 3 times as long as it is wide, and its area is 2883 sq. rd. What are its dimensions?

24. A triangle whose base equals its height has an area of 10,082 sq. in. What is the length of the base?

25. A triangle whose base equals twice its height has an area of 5041 sq. in. Required the base and height.

26. A rectangular field is 4 times as long as it is wide, and its area is 2500 sq. rd. What are its dimensions?

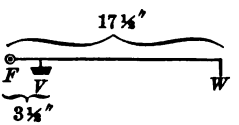
27. The base of a certain triangle is 4% more than the altitude. The area is 74.88 sq. in. Required the base.

28. The surface of a sphere is approximately $12\frac{4}{7}$ times the square of its radius. What is the radius of a sphere whose surface has an area of 616 sq. in.?

29. At a school entertainment the price of the tickets the second day was reduced 25%, which resulted in an increase of receipts by 8%. What was the per cent of increase in the number of tickets sold?

30. Albany and Buffalo are 297 mi. apart. If a train leaves Albany for Buffalo at 6 A.M. and travels 35 mi. an hour, and at the same time one leaves Buffalo for Albany and travels 45 mi. an hour, without stopping, how far apart will they be in one hour? When will they meet?

31. A safety valve is $2\frac{1}{2}''$ in diameter where it comes in contact with the steam, and is $3\frac{1}{2}''$ from the fulcrum F. The lever is $17\frac{1}{2}''$ long. A weight is to be hung at W that shall let the steam blow off at 90 lb. pressure to the square inch. The formula for the safety valve is $\pi r^2 pd = Wd'$, where r is the radius of the valve, p is the pressure per square inch, d is the distance FV, W is the weight, and d' is the distance FW. Find the value of W, using $3\frac{1}{7}$ for π.



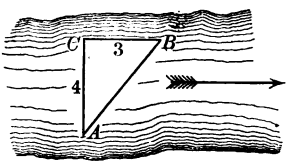
32. A boy holds a foot ruler vertically at such a distance from his eye that he can see the top of the school building just in a line with the top of the ruler, and the bottom of the building just in a line with the bottom of the ruler. The distance of the bottom of the ruler from his eye is then measured and found to be 1 ft. 3 in., and the distance from the boy's eye to the bottom of the building is 100 ft. What is the height of the building? Draw a diagram.

33. How long is the diagonal of a hall a ft. by b ft.? of a carriage house 10 ft. by 24 ft.?

34. What is the direct distance from the top of a 100-foot tower to a spot 75 ft. from the base?

35. A school flagpole is broken by the wind 16 ft. from the ground. The two pieces hold together, and the top of the pole touches the ground 30 ft. from the base. Find the length of the pole.

36. If I start to row directly across a stream, in the direction AC, at the rate of 4 mi. an hour, and if the stream carries me in the direction CB at the rate of 3 mi. an hour, my course will really be AB, as a result of these two motions. What is my rate of progress?



37. Suppose I row at the rate of 4.5 mi. per hour, and the stream flows 6 mi., what is my rate of progress?

38. If the diamond of a baseball field is a square 90 ft. on a side, how far is it from first base directly across to third? (Two decimals in this and similar examples.)

39. If the top of a stepladder is $4\frac{1}{2}$ ft. above the floor, and the bottom is $3\frac{1}{2}$ ft. from a perpendicular drawn from the top to the floor, how long is the ladder?

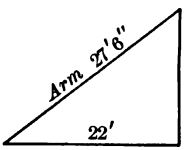
40. From a point on a wall $6\frac{1}{2}$ ft. above the floor a string is stretched to a point on the floor 9 ft. from the wall. How long is the string?

41. The side of one square is 31 in. and that of another is 14 in. What is the side of a square that is equal to the sum of these squares?

42. If the sum of two squares is 24 sq. in. and one of the squares is 14 sq. in., what is the side of the other square?

43. A telegraph pole is set perpendicular to the ground. In order to hold it firmly, one end of a wire is fastened to the pole 18 ft. from the ground, and the other end of the wire is fastened to a stake at the level of the ground 13 ft. 6 in. from the foot of the pole. How long is the wire?

44. A derrick for hoisting coal has its arm 27 ft. 6 in. long. It swings over an opening 22 ft. from the base of the arm. How far is the top of the arm above the opening?



45. How long will it take a sum of money to double itself at 6% simple interest? at 5%? at 4%?

46. At what per cent will a sum of money double itself in $16\frac{2}{3}$ years at simple interest? in $18\frac{2}{11}$ years? in $19\frac{1}{21}$ years?

47. A dealer sold 500 barrels of flour at a discount of 5%, receiving $2422.50. What was the list price per barrel?

48. A freight train which travels 18 mi. an hour is followed after 4 hours by a passenger train which travels 24 mi. an hour. In how many hours will the passenger train overtake the freight train?

49. A train which travels 32 mi. an hour is 45 min. ahead of a train which travels 42 mi. an hour. In how many hours will the latter overtake the former?

50. Two men start from points 28 mi. apart and walk toward each other, the first at the rate of 4 mi. per hour and the second at the rate of 3 mi. per hour. How far from where the first man started will they meet?

51. Under favorable summer conditions sound travels 1125 ft. per second. If a gun is fired from a warship 2 mi. away, how long after the flash should the sound be heard?

52. If s times the weight of a cubic foot of water is the weight of a cubic foot of iron, we say that the *specific gravity* (or density) of iron is s. Find the value of s if a cubic foot of water weighs $62\frac{1}{2}$ lb. and a cubic foot of iron weighs 450 lb.

53. From Ex. 52, if a cubic foot of steel weighs $487\frac{1}{2}$ lb., what is the specific gravity of steel?

54. From Ex. 52, if a cubic foot of ice weighs $57\frac{1}{2}$ lb., what is the specific gravity of ice?

55. There are two cylindrical tanks with equal depths. Their diameters are respectively 24 ft. and 10 ft. Find the diameter of a tank of the same depth and holding as much as the two together. (The square on the required diameter equals the sum of the squares on the other two.)

56. Two branches of an iron water pipe are respectively $1\frac{1}{2}$ in. and 2 in. in diameter. Find the diameter of a pipe that shall just carry away the water from both branches.

57. A manufacturer is building a chimney with a circular flue for his boiler room. He has two horizontal boilers, each containing 96 tubes 2 in. in diameter. His chimney must have the same cross-section area as the sum of all the cross-section areas of the tubes of the boilers. Required the diameter of the flue.

58. In shipping certain goods the net rate after deducting 15% from the usual rate is 34¢ per hundred pounds. What is the usual rate?

59. A man had three notes for equal amounts, the first drawing 6%, the second 5%, and the third 4%. The interest on all three amounted to $112.50 a year. What was the face of each note?

60. The wooden pattern from which an iron casting is made weighs $6\frac{1}{4}$% as much as the iron. If the pattern weighs $31\frac{1}{4}$ lb., how much does the casting weigh? If the pattern weighs $62\frac{1}{2}$ lb., how much does the casting weigh?

61. Find in inches the depth of a cylindrical tank 5 ft. in diameter that has the same capacity as a rectangular cistern 8 ft. square and 6 ft. deep.

62. A farmer is planning to build a poultry house for 350 fowls. He estimates the average weight per fowl to be 6 lb., and he allows 10 cu. ft. of air space to each pound of fowl. The house is to be 16 ft. wide and to have an average height of 8 ft. How long must he build the house in order to give it the required capacity?

63. If an emery wheel makes 2000 revolutions per minute, its grinding speed per minute is 2000 times the circumference. If the diameter is 10 in., what is the grinding speed?

64. A flywheel of which the diameter is 6 ft. makes 100 revolutions per minute. At what rate per minute does a point on the rim travel?

65. How many revolutions per minute must be made by a circular saw the diameter of which is 28 in., in order that the saw may have a cutting speed of 8800 ft. per minute?

66. A room is 16 ft. long, 12 ft. wide, and 9 ft. high. How far is it from an upper corner diagonally through the room to the opposite lower corner?

67. In how many years will the principal and interest on \$5000 at 6% equal the principal and interest on \$6000 at 3%?

68. A builder sold a house for \$5600, thereby gaining 12% on what the house cost him. How much did the house cost him, and how much did he gain?

69. A man pays \$25,000 for a factory, borrowing the money at 5%. He allows 1.8% for taxes and 2.4% for insurance, depreciation, and repairs, and rents it for \$3750 a year. What is the net rate of income?

70. A man holds one mortgage of \$2500 and another of \$3000. The rate of interest on one is 1% more than on the other, and the income is the same on each. What is the rate of income on each?

71. A man was in arrears an equal amount with his interest on two mortgages, one mortgage being \$1500 more than the other. On the first one, bearing 6%, he was three years in arrears, and on the second, bearing 5%, two years. What was the face of each mortgage?

72. The circumference of a fore wheel of a carriage is 9 ft., and that of a hind wheel is 12 ft. How far has the carriage traveled when the fore wheel has made one more revolution than the hind wheel?

73. The circumference of a fore wheel of a carriage is c feet, and that of a hind wheel is C feet. Find the distance that the carriage has traveled when the fore wheel has made n revolutions more than the hind wheel.

74. A sum of money at simple interest amounted in t years to a dollars, and in t' years to a' dollars. Find the sum and the rate of interest. From this formula find the sum and rate when $a = 236$, $t = 3$, $a' = 260$, and $t' = 5$.

75. From Ex. 74 find the sum when $a = 2118.30$, $t = \frac{7}{12}$, $a' = 2139$, and $t' = \frac{5}{6}$.

JUN 8 1918